VOLUME XX[B] SEPTEMBER, 1928 PART II

AN INVESTIGATION OF THE MILK YIELD OF DAIRY COWS

BEING A STATISTICAL ANALYSIS OF THE DATA OF THE SCOTTISH MILK RECORDS ASSOCIATION FOR THE YEARS 1908, 1909, 1911, 1912, 1920 AND 1923.

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I. Introductory.

(1) *Origin of the Investigation.*

The advantages of ascertaining the milk yield and of determining the quality of milk, together with the recording of the same in a systematic manner, have been recognised in this country for many years. The first attempt at systematic recording was made under the auspices of the Highland and Agricultural Society of Scotland in 1903. Milk recording was fostered and encouraged by the Society, and several interesting reports, prepared by the late Mr John Spier, Newton, Ayrshire, appeared annually in the *Transactions* up to 1908. Towards the end of 1907 the Society thought it desirable that the work of recording should be more directly in the hands of the dairy farmers themselves, and accordingly the work was transferred to a body called "The Ayrshire Cattle Milk Records Committee," under whose supervision the tests were conducted for several years. In 1910, the name of the Committee was altered to "The Scottish Milk Records Committee," and finally, on the 28th February, 1914, at a meeting held in Kilmarnock, the administration of milk recording in Scotland was formally taken over by a newly created body designated "The Scottish Milk Records Association." This body now consists of over forty local societies, and during 1926 made observations upon 29,236 dairy cows. Milk recording in a systematic manner has thus been in existence in Scotland for the past 24 years and, up to

1926, observations had been made and recorded on 417,739 cows. The suggestion that valuable scientific results might be obtained by a systematic analysis of these records was first made by the late Dr C. M. Douglas, C.B., of Auchlochan, at an ordinary meeting of the governing body of the West of Scotland College of Agriculture held in Glasgow on 30th June, 1913. The Governors favourably considered the proposal, and it was forthwith agreed to make an application to the Development Commissioners for a grant to meet the cost of a statistical analysis during 1913–14. The application was duly made through the Scottish Board of Agriculture, and in August of the same year the Board replied that they were prepared, on the recommendation of the Advisory Committee on Agricultural Science, to make a grant not exceeding £200 for the ensuing year. Soon after this a special committee was appointed to carry out the decision of the governing body. The writer was appointed by this committee to conduct a statistical investigation of the data and in particular to examine the data for the purpose of elucidating the following points, namely:

I. Relation of quantity and quality, butter fat ratios for yields of five hundred gallons, six hundred gallons, seven hundred gallons, eight hundred gallons, nine hundred gallons, and one thousand gallons and over.

II. Relation of quantity and quality to age of cow; average of (*a*) butter fat ratio, (*b*) quantity of yields of cows aged respectively two years, three years, four years, five years, six years, seven years, eight years, nine years, ten years, and over ten years.

III. Relation of yield to date of calving, average yields of cows calving in each month of the year.

IV. Comparison of selected districts of (*a*) quantity and (*b*) butter fat ratio*.

V. Another point bearing on the influence of sire and dam respectively upon lactation was included in the reference, but owing to the magnitude of the foregoing was, later on, omitted from the present investigation.

It was understood that, while attempting to answer the foregoing questions, the writer was left free to attack from time to time such problems as appeared to him necessary from the nature of the data.

The writer desires to express his indebtedness to the Board of Agriculture for Scotland for meeting part of the cost of the investigation by grants from the Development Commissioners. I have had the valuable assistance in the collation and analysis of the data, of successive workers in my laboratory in the following order, namely, Mr George Rae, B.Sc.; Dr W. O. Kermack; Mr J. S. Thomson, M.A., B.Sc.; Miss Clara Esslemont; Mr A. B. Thomson, M.A.; and Miss M. H. Anderson, B.Sc. I have also to acknowledge the able assistance throughout the investigation of Mr John E. Ritchie, M.A., B.Sc., F.I.C., of my analytical staff.

* The results of the analysis of the data with respect to districts are given in the *Transactions of the Highland and Agricultural Society of Scotland*, 1919. Investigation into the milk yield of Ayrshire cows, pp. 237–56. Only classes I and II were studied.

(2) *The Records for* 1911 *and* 1912.

The Committee supplied from time to time the Records of the Association for the years 1908 to 1912. Each farm, although not named in the records, was indicated by a number, as was also each individual cow in the herd. The observations made on each cow and noted in the records were found to be as follows:

(i) Estimated Total Quantity of Milk in gallons for the entire Lactation Period.

(ii) Average percentage of Butter Fat yielded during the Lactation Period.

(iii) Length of Lactation Period in weeks.

(iv) Age of Cow in years.

(v) Date of Calving (beginning of record).

(vi) Date of second Calving (beginning of next record).

(vii) Remarks.

It was found that the records could be divided into three classes according to their degrees of completeness, namely:

(*a*) Cases in which dates of birth of calves were given both at opening of lactation period and its close (i.e., beginning of next lactation period).

(*b*) Cases giving a lactation period but lacking in the date of second calving.

(*c*) Cases of evidently incomplete lactation periods.

After a preliminary examination of the entire data submitted to the writer, a report was submitted to the Committee. Instructions were then given by the Committee to use, in the complete statistical analysis, only those records which were complete in every way for the purpose of the investigation, namely, those contained in Class (*a*) above. It was also decided to confine the present research to the records belonging to the years 1911 and 1912.

Cows were found to be classified by the local Recording Societies constituting the Association into the following six classes*:

I. Cows yielding over 2500 gallons of milk calculated as containing one per cent. of butter fat = 250 pounds.

II. Heifers yielding over 2000 gallons of milk calculated as containing one per cent. of butter fat = 200 pounds.

III. Cows yielding under 1660 gallons of milk calculated as containing one per cent. of butter fat = 166 pounds.

IV. Heifers yielding under 1330 gallons of milk calculated as containing one per cent. of butter fat = 133 pounds.

* The Scottish Milk Records Association from 1924 onwards prints a Register of Class I Cows, Class I being now defined as cows with a milk yield equivalent to not less than 2800 gallons containing one per cent. of fat, equal to 280 pounds, the corresponding figure for heifers being 2240 gallons, equal to 224 pounds.

V. Cows yielding between 1660 and 2500 gallons of milk calculated as containing one per cent. of butter fat.

VI. Heifers yielding between 1330 and 2000 gallons of milk calculated as containing one per cent. of butter fat.

It will be useful at this stage to give a brief description of the methods employed by the officials of the Association in collecting the data. The work of noting the quantities of morning and evening milk and the percentages of butter fat daily obtained from each cow, and the relative expense, were so great as to preclude the Milk Records Association from making daily records. The morning and evening milk of each cow in each herd was tested at intervals ranging from 14 to 28 days. The operation was repeated after an interval of about fourteen days. The yield of milk and the percentage of butter fat yielded by each cow in the interval were assumed to be those given on the date of the previous test. The unit employed was that which reckoned the yields in terms of gallons of milk containing one per cent. of butter fat. This really amounted to recording the total amount of butter fat yielded by each cow during the specified interval, the yield being based upon one determination of butter fat and one determination of yield of milk during that interval. The milk was weighed and recorded in pounds, and in the records the term "gallon" is applied to the weight of ten pounds of milk, and has therefore a special meaning, and is not the usual gallon measure. As an example of recording, suppose thirty pounds was the yield of milk from one cow on a particular date and that this cow was again tested after an interval of fourteen days. Suppose also that the milk from this cow was found to contain 3·5 per cent. of butter fat. The recorder converted the thirty pounds of milk containing 3·5 per cent. of butter fat into pounds of milk containing one per cent. of butter fat, or $30 \times 3{\cdot}5 = 105$ pounds of milk one per cent. quality. The recorder then multiplied the pounds of milk of one per cent. quality—one day's yield—by fourteen and thus provided an estimate of the number of pounds of milk of one per cent. quality likely to be obtained during the fourteen days between the two tests, that is, $105 \times 14 = 1470$ pounds of milk of one per cent. quality in fourteen days, or 14·7 pounds of butter fat in fourteen days. This procedure was repeated on every occasion a test was made, and at the end of the lactation period (that is, the whole period between two calvings) of the cow the total estimated yield of milk of one per cent. quality was ascertained by summation. The average percentage of butter fat for each cow for the whole lactation period was found by the recorder by dividing the number of pounds of milk of one per cent. quality by the total number of pounds of milk. In printing the selected records, the Association recorded the total estimated yield of milk for each cow in gallons, the gallon being as already stated, the weight of ten pounds and not a measure. The yield of milk of one per cent. quality was entered in each case in the written records but was not given in the printed records. It is thus clear that the yield of milk determined by the Scottish Milk Records Association during the entire lactation period is not the *actual* yield directly ascertained day by day, but an *estimated* yield based on periodic determi-

nations. The same is true of the average of butter fat recorded; it is not the real average ascertained from daily determinations of butter fat, but an estimated average based on periodic, mainly fortnightly determinations.

In order to make the statistical analysis, the data bearing on each cow was placed on a separate card. Each card showed for the particular cow: (1) the total estimated yield of milk in gallons, the term gallon being, as already stated, a quantity of milk weighing ten pounds; (2) the average quality of the milk as indicated by the percentage of the butter fat; (3) the age of the cow; (4) the duration of the lactation period in weeks, and (5) the total quantity of butter fat in pounds, as calculated from the quantity and quality of milk for the whole lactation period.

Each card thus showed the magnitude, for each particular cow, of the five "characters" investigated, viz.:

(1) Quantity of milk in gallons. (*Q.*)
(2) Percentage of butter fat. (*P.*)
(3) Age of cow. (*A.*)
(4) Weeks in milk (duration of lactation period). (*W.*)
(5) Yield of butter (total quantity of butter fat). (*B.*)

The whole of the data having thus been transferred to cards, these latter were then arranged in order to obtain frequency distributions for the five characters so as to investigate the variation in each, and to make correlation tables from which to calculate the relationship existing between any two characters. For each character the cards were sorted into classes, and the following sub-ranges were decided upon in the various cases.

(1) *Quantity of Milk*—a sub-range of 50 gallons.

All cows showing a quantity from 150 to 200 gallons inclusive were placed in one class, with class centre at 175 gallons, those showing a quantity from 200 to 250 gallons in a second class with class centre at 225 gallons, and similarly from the remaining classes.

(2) *Percentage of Butter Fat*—a sub-range of 0·25 per cent.

All cows showing an average percentage of butter fat from 2·00 to 2·25 inclusive were placed in one class with 2·125 as the centre of sub-range, those showing a percentage of from 2·25 to 2·50 inclusive in a second class, with class centre at 2·375, and similarly for the remaining classes.

(3) *Age of Cow.* In this case each year of age was made a separate class from 2 years up to 18 years, the last class containing cows of 18 years and over. Cows over x years and less than $(x+1)$ years were entered by the recorders as aged x years. Hence the centre of the class ranging from over x to less than $(x+1)$ years was $(x+\frac{1}{2})$.

(4) *Weeks in Milk*—a sub-range of three weeks.

All cows showing from 7 to 9 weeks inclusive were placed in one class with 8 weeks as the class centre; those of from 10 to 12 weeks inclusive in a second class with 11 weeks as the class centre, and similarly for the remaining classes.

(5) *Yield of Butter Fat*—a sub-range of 20 pounds.

All cows showing 60 to 80 pounds inclusive were placed in the first class, the centre of the class being 70 lbs., all cards showing 80 to 100 pounds inclusive in a second class, with the class centre at 90 pounds, and similarly for successive classes. Butter fat yield was determined to the nearest half-pound. Yields of $(x-\frac{1}{2})$ upwards and less than $(x+\frac{1}{2})$ were placed in the x sub-class. The cards were also arranged according to the various local milk records societies to which they belonged, in order to ascertain the average yield of milk and the average quality (percentage of butter fat) for each local society.

(3) *The Records for* 1920.

On the completion of tne work on the 1911–12 data, the Scottish Milk Records Committee had under consideration what further statistical analyses on the Scottish data should be made, in order to throw light on the value of recording as revealed by the annual variations in the characters recorded, and in the annual changes in the relationships between the various recorded characters. The following is a brief summary of the nature of the research as outlined by the committee.

(1) In any statistical analysis which may be undertaken on these records it is desirable that the analysis should not be made upon the printed data but upon the written records. The analysis, however, should be confined to these records where the date of birth of calf at the end of milk record has been recorded as well as date of birth of calf at the beginning of record. Incomplete lactation periods should be omitted and also cases where there is no record of a second calf being born within sixty weeks. The analysis would then include cases where the complete lactation period is known and is approximately normal.

(2) Measurement of Annual Variations in Yield and other Characters.

In order to determine the annual variations in yield, percentage of fat, duration of lactation period, total yield of butter fat, and age of cow, the data for each year from 1903 to 1920 should be extracted and analysed. This means the tabling of the records of complete lactations on cards for the purpose of collation and analysis. As a final result a table should be prepared showing the annual values from 1903 to 1920 of the recorded characters.

This work includes the measurement of the variations for these characters in order to determine the significance of the difference of the value of each character for every year. A final table and a series of graphs would then be prepared showing the change whether progressive or retrogressive.

(3) Yield and Quality.

In order to determine how far for each year an increase in yield is accompanied by a decrease in quality, a series of correlation tables would have to be prepared for these characters for each of the years. On collating and analysing the data we should then be able to prepare a table showing whether the relationship between these characters is constant, or whether it has varied with the selection or change in the breed.

(4) Correlations.

In order to determine how far the other factors are correlated, a series of correlation tables should be prepared for each pair of characters for each of the years studied. A table should then be prepared showing the relationships between the factors.

(5) Monthly Classification.

With reference to the characters proposed to be investigated, it was agreed that a further classification should be made according to the month in which the lactation period begins.

(6) Private Daily Records compared with Official Tests.

It was further agreed that certain daily records should be compared with the results of fortnightly, three-weekly, and monthly tests, in order to determine the relative accuracy of these.

(7) It was agreed that, in the first instance, the enquiry should deal with the years 1908 and 1920, and that the inclusion of other years should be considered later.

On communication with the Superintendent of the Scottish Milk Records Association, the writer learned that he had not in his possession the complete summary sheets prior to the year 1911. The writer had, therefore, to abandon the idea of conducting a statistical analysis on the 1908 records, and to concentrate upon the 1920 records, the summary sheets of which he received from the superintendent from time to time. The data from these sheets were transcribed onto cards, an illustration of which is shown in Table I.

TABLE I.

1920 *Records.*

		No. of Society.		
	Farm Letter.	Breed.	No. of Cow.	
	Total Yield	Percentage	Yield at 1 %	Weeks in
Age	in Gallons.	Butter Fat.	Butter Fat.	Milk.
	Date of Opening of Record.		Date of Closing of Record.	
	Date of Calving.		Date of Next Calving.	
	Calculated Yield per week in Gallons.		Calculated Yield of Butter Fat per week in Pounds.	

The total number of cows tested during 1920, as noted from the records, amounted to 24,023. In 2573 cases the age and other particulars were found to be wanting. The records therefore of 21,450 were collated. Of these, 5320 were heifers and 16,130 were cows. Of the total number of cows 14,416 were Ayrshires and 1714 were cows belonging to various breeds including British Friesian, Aberdeen Angus, Irish Crosses and Shorthorn Crosses. The following table (Table II) shows the numbers of Ayrshires and other breeds.

TABLE II.

	Cows	Heifers
Ayrshires	14,416	4,776
Other Breeds	1,714	544
Total	16,130	5,320

Total cows and heifers 21,450.

There were found to be 14,416 Ayrshire cows ranging from 3 to 20 years of age. Of these, it was found that approximately 6500 had no record of a second calf, about 1000 did not give birth to a second calf until after 60 weeks from the previous calving, while about 2000 cards showed "closing of record" in December, 1920, but were evidently cows whose lactation periods ran well into 1921. After eliminating these we were left with a total of 4912 Ayrshire cows whose lactation periods were completed in 1920 and with a record of a second calf within 60 weeks of the previous calving.

(4) *Previous Work.*

A study of the records kept at Lord Rayleigh's dairy farms, Terling, Essex, was made by W. Gavin*. He studied the relationship between (*a*) the maximum daily yield maintained or exceeded for not less than three entries in the record book, and (*b*) the total yield during the lactation period. He showed that the correlation between these two factors was in the neighbourhood of 0·85 and concluded that the total milk yield of a cow for a lactation period can usually be judged, within a few weeks of calving, from the lowest figure of the three highest entries for daily yield. The figure thus obtained he calls the "revised maximum" (R.M.). The regression of R.M. on total yield was found to be linear. The influence of number of calvings on yield (R.M.) and the influence of date of calving on yield (R.M.) were also studied by this writer. He found that the highest R.M. occurred in April and May, when there is an abundance of natural succulent food.

In his "Studies in Milk Records" Gavin showed that the yield during the first lactation period was more variable than the yields in succeeding lactation periods. He suggested that among the total yields during a lactation period of a cow, the maximum total yield should be used to represent her mature capability. Gavin concluded that the R.M. of the first lactation period should be used to determine whether a cow should be kept for dairy purposes or not. In doubtful cases he recommended an estimate based on the mean R.M. for the first and second lactation periods.

In a paper by Eckles and Palmer† it is suggested, but is not statistically established, that the average percentage of fat in the milk of Jersey cows attains a maximum during one of the first three lactation periods.

Pearl and Miner‡ have made a study of the variation in the quantity and in the fat-content of milk of Ayrshire cows from the records of the Ayrshire Cattle

* "The Interpretation of Milk Records," *Journ. Roy. Agric. Soc.* 1912, p. 153; and *Journ. of Agric. Science*, 1913, Vol. v. Part 4, "Studies in Milk Records; On the Accuracy of Estimating a Cow's Milking Capability by her First Lactation Yield."

† *Journal of Agricultural Research*, Vol. II. No. 12, 1917, pp. 645—657.

‡ *Journal of Agricultural Research*, Vol. XVII. No. 6, Sept. 1919, pp. 285—322.

Milk Records Committee for the years 1908 and 1909. The investigation was confined to those records of cows which had been 32 weeks or more in milk. The frequency distributions of quantity and of fat content of the milk of cows of the same age were found for each year of age. The authors found that these distributions, when fitted with Pearsonian curves, belonged to Types I, II, III and IV. They give no suggestion, however, as to the area on the β_1, β_2 plane to which the frequency is restricted, which, after all, is the important fact we want to know in considering the physical characters of observed distributions. Normal curves were also found to fit certain of the distributions. Regression curves of mean weekly yield on age and of mean fat content on age were fitted to the data. The work of these authors is referred to in Section II and Section III, where the results are compared with the writer's figures for the years 1911, 1912 and 1920.

J. W. Gowen has discussed the conformation of Jersey cattle and its relation to milk producing capacity*. He showed that the correlation between the udder and milk yield (0·1906) was practically as great as the correlation between (1) the characters selected by him as constituting "conformation" and (2) milk yield (0·1941).

A further memoir has been published by J. W. Gowen†, giving the results of his analyses of the records of a Jersey herd over a number of years. The size of the herd is, however, not stated, but it is stated that the records of the *same* cows over a number of years have been studied. The author extracted 1741 complete eight months' records of healthy cows for analysis. How many cows the 1741 records represent is not stated and the reader should therefore note that the results are not from 1741 *different* cows. Gowen studied his data with respect to age and other factors. He included all cows of 10 years of age and upwards in his highest age group. He found that Pearsonian curves of Types I, II, and IV prevailed for the yield distributions and found good fits for these curves. Gowen found the correlation between age and yield to be significant and the regression to be non-linear. He showed that the variability in milk yield increased with age up to about 7 years, after which the variability steadily decreased. In technical language, milk production with age is heteroscedastic. The author showed, for a group of cows between 2 and 3 years old, the mean yield for a lactation period of 8 months. The same group was followed until the cows were over 10 years of age and he showed that there was a progressive increase in yield for the *same* cows for every year of age up to 7 years, after which there was a decrease in mean yield with age. The variability of milk yield was similarly shown to increase uniformly up to nearly 7 years when the variability declined with age. In other words, the scedastic curve was linear until it reached its maximum, about 7 years of age. There was found to be no very great change in the coefficient of variation for milk production of a cow as it gets older, nor can it be said that the coefficients of correlation between the 2 year old group and each of the older groups were significantly different among themselves.

* *Journal of Dairy Science*, Vol. III. No. 1, Jan. 1920, pp. 1—32.

† *Genetics*, Vol. v. No. 2, March 1920, pp. 111—188.

From records of the same cow throughout, the author showed that from 2 to 3 years of age she contributed less than her expected yield towards the total for five lactations, while, at from 3 to 4 years of age, she contributed slightly more than her expected yield. At other ages she produced the expected proportions. The author pointed out, from the results, that the milk production of one lactation was highly correlated with the total milk production of two lactations of which the given lactation is one component, and therefore the milk production of one lactation was the determining factor for the total milk production of two such lactations.

Gowen, in a further contribution, gave the results of a study of the correlations between fat percentage and age in Jersey cattle*. He showed the range of variation in butter fat percentage to be from 3·65 to 6·95. Curves of Types I, III, IV and V and normal curves were fitted to the distributions of butter fat. The regression of mean percentage of butter fat on age for a lactation of eight months was found to be linear. The curve of the standard deviations of butter fat on age was also linear, and there was a very slight diminution of variability with age. The Jersey cow tended to give a less percentage of butter fat during the eight months' lactation period as she got older. The mean percentage of butter fat was 5·23, with a corresponding standard deviation of 0·449 and a coefficient of variation of 8·6. The author concluded that the relative accuracy in the use of one lactation period to predict the expected record of another lactation period was approximately the same for percentage of butter fat as for milk yield. He also showed that the butter fat percentage of one lactation period was highly correlated with the butter fat percentage for the first five lactations (0·78 to 0·86). He held that the factors which govern butter fat percentage have their power maintained in the same relative strength throughout the life of the cow to the exclusion of any group of short period factors.

II. Study of the 1911 and 1912 Records and Comparisons with other Data.

(1) *Average Yields and Average Values generally.*

The mean or average value of each of the characters studied was calculated for the two years 1911 and 1912, and the results are given in the following table (Table III).

TABLE III.

Character	1911 Data (Total 4591)	1912 Data (Total 6648)
Quantity of Milk	634·82	654·20 gallons 643·26 gallons†
Percentage of Butter Fat	3·73	3·74 per cent.
Weeks in Milk	36·78	37·41 weeks
Yield of Butter Fat	236·13	243·76 pounds
Age of Cow	6·31	6·19 years

* *Genetics*, Vol. v. No. 3, May 1920, pp. 249—324.

† Average yield for 1912 reduced to a constant lactation period of 36·78 weeks (the average for 1911).

In 1911, the average yield was thus about 20 gallons less than in 1912, while the quality (percentage of butter fat) remained practically constant. The average length of a lactation period and the average age of the cows show but a slight variation from the one year to the other.

The average total yield of butter fat amounted to 236 pounds in 1911 and 244 pounds in 1912. This corresponds to a yield of butter (containing the maximum amount of water permissible under the Food and Drugs Acts, viz. 16 per cent.) of 281·0 pounds in 1911 and 290·5 pounds in 1912.

In order to ascertain whether or not there is a significant difference between these averages for the two years, the probable error of the difference was calculated in each case, the results being shown in the following table (Table IV).

TABLE IV.

Significance of Differences between Means.

(m_1, σ_1, N_1 refer to 1912) (m_2, σ_2, N_2 refer to 1911)

Character	$(m_1 - m_2)$	$\cdot 67449 \sqrt{\sigma_1^2 + \frac{\sigma_2^2}{N_2}} = E$ *	$\frac{(m_1 - m_2)}{E_{(m_1 - m_2)}}$
Quantity of Milk (gallons)	19·38	2·1264	9·11
Quantity of Milk for constant Lactation Period	8·44	2·1264	3·97
Percentage of Butter Fat	0·01	0·0044	1·83
Weeks in Milk	0·63	0·0836	7·49
Total Yield of Butter Fat (lbs.)	7·64	0·7972	9·59
Age of Cow (years)	0·12	0·0351	3·37

* Some of the cows will be the same for the two years, but as the correlation between the milk from the same cow in two successive years is positive, this formula gives a maximum value and therefore the values in last column will be minimum values.

It is thus evident that there is a significantly larger yield of milk and also of total yield of butter fat in 1912 than in 1911; but, as the lactation period is also significantly longer, it is interesting to consider whether this is the cause of the greater yield. The average quantity of milk for 1912 when reduced to a lactation period of 36·78 weeks would be 643·26 gallons, the difference between this and the 1911 yield being thus reduced to 8·44 gallons which, when divided by the probable error of the difference, viz. 2·1264, gives 3·97. This would indicate that there is probably a significant difference in the quantity of milk apart from the longer lactation period in 1912. Such an increase might be expected from year to year, within limits, owing to the retention of cows giving good yields and the elimination of those giving poor yields. The difference in yield between 1911 and 1912 shows that a characteristic value for each year is likely to exist, and that there is likely to be an increase in the average yield. The difference between the average percentages of butter fat for 1911 and 1912, does not appear to be significant.

(2) *Frequency Distributions.*

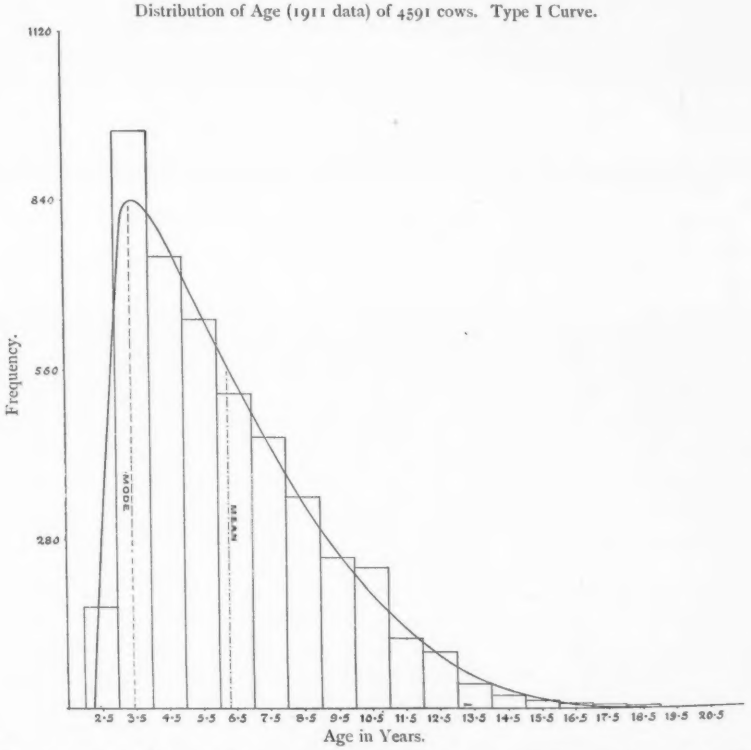
We shall now consider the nature of the distributions of the various characters for 1911 and 1912.

(*a*) *Age of Cow.*

The unit of analysis for the age distribution was one year. The mean age of the 1911 series was 5·81 years, while the corresponding value for the 1912 series was 5·69. Naturally the age distributions exhibit high degrees of positive asymmetry, the values of Sk/E_{sk} (85 and 83 respectively) being far greater for age than for any of the other characters. The criterion, κ_2, is negative and indicates one of the group of curves belonging to Type I (see Diagrams I and II). The mode for 1911 is at

DIAGRAM I.

Distribution of Age (1911 data) of 4591 cows. Type I Curve.

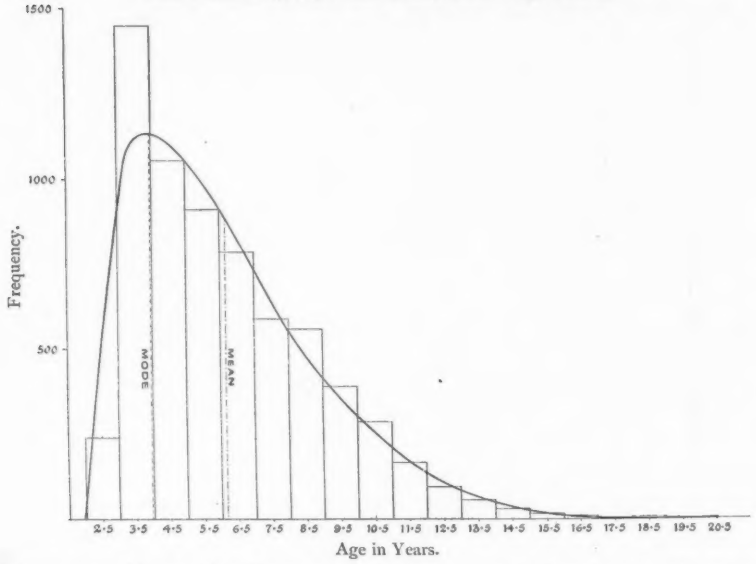


3·47 years and, therefore, d is equal to 2·84. The corresponding value of d for 1912 is 2·24. The general practice of dairymen in the Association is to reject animals found on trial to be comparatively poor milkers. Thus, we have greater observed frequencies among the cows 3·5 years of age than the theoretical frequencies obtained by graduating the series with Pearsonian curves.

It is evident from a study of the combined distribution (1911 + 1912), that the frequency at 3·5 years of age is too high to allow of the Type I curve being a good fit. It appears evident that about 350 cows included in the three year old class should really be in the two year old class. In 1911 and 1912, the practice with regard to recording the age of cow was not so precise as the present practice when the age of the animal in years and months is given. It is usual for dairymen to arrange the date of calving to suit their purposes. For instance, cheese-makers usually arrange to have their cows calving about the month of February. In the records the few animals that are over, say, two years of age and less than three, are reckoned as 'two year olds,' and all animals that are over three and have not reached four are reckoned

DIAGRAM II.

Distribution of Age (1912 data) of 6648 Cows. Type I Curve.

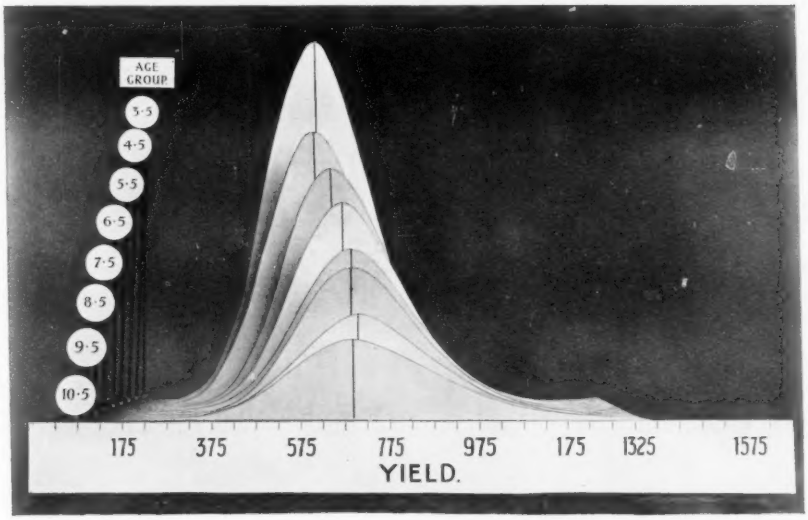


as three year old animals. There are cases, however, of heifers having calves before they reach three years of age and they would be reckoned (having had a calf) as being three year old heifers. There is a distinct drop in the number of cows after eleven years of age in the observed frequencies when compared with the theoretical curve.

(*b*) *Quantity and other Characters for all Ages.*

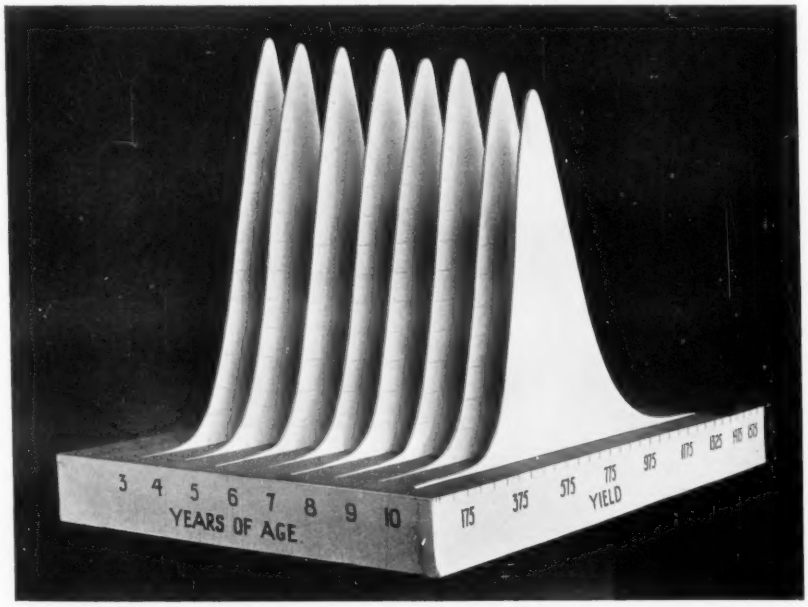
As can be seen from the regression curves of Quantity on Age, Quantity is a function of Age. The distributions of Quantity for all ages will therefore be a heterogeneous distribution. Theoretical curves, although they can have no physical meaning, were fitted to the data in order to see what kind of frequency distributions were indicated, and also to see whether the curves reasonably described the data. The values of the criterion for both years were found to be between 0 and 1. The

DIAGRAM III.



Photograph from card model of Frequency of Total Milk Yield for a Lactation Period of Cows of each Age.

DIAGRAM IV.



Photograph from a second card model of Frequency of Total Milk Yield for a Lactation Period of 100 Cows of each Age, *i.e.*, Percentage of Cows at each Age with a given Yield.

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distributions are consequently skew with unlimited range in both directions and are therefore Type IV curves. The values of σ^2, β_1 and β_2 are given in the table below (Table V). Frequency curves were also fitted to *P*, *W* and *B* in order to see whether these curves also reasonably described the data. The fits were found to be bad also, owing to the heterogeneity of the material with respect to age.

(c) *Distribution of Quantity for different Ages.*

In consequence of the fact that the distribution of Quantity for all ages are heterogeneous distributions, it was found necessary to fit curves to the distribution of quantity for each year of age from 2·5 years onwards. For this purpose the data for both 1911 and 1912 were taken together and statistically examined. Upon analysis it was found that these distributions were adequately fitted by Type IV

TABLE V.

Analytical Constants of Curves.

1911 *Data.*

Character	No.	Unit	σ^2	β_1	β_2	*P*
Q	4671	50 gals.	9·9232	·2418	3·5655	·209
P	4671	·25 %	1·6782	·1489	3·2964	·473
A	4591	1 year	7·5117	·9140	3·6061	·015
W *	4557	3 weeks	3·5877	·0424	3·7103	·001
B	4591	20 lbs.	8·7896	·3257	3·7961	<·001

* Distribution up to 56 weeks.

1912 *Data.*

Character	No.	Unit	σ^2	β_1	β_2	*P*
Q	6648	50 gals.	11·7517	·1679	3·4867	·001
P	6648	·25 %	1·8431	·0794	3·2988	·239
A	6648	1 year	7·0861	·8352	3·5090	very small
W	6648	3 weeks	4·9454	·4090	6·0834	very small
B	6648	20 lbs.	10·2831	·1794	3·5184	·050

curves, the frequency constants of which are given in the undernoted table (Table VI). It will be seen from the table and from Diagrams III and IV that the mean yields increase with increasing age until the 10·5 age group is reached, while the modal yields increase until the 9·5 age group is reached, after which there is a slight fall in both characters. The heifer group (2·5 age group) has a relatively high variability. The variability falls for the 3·5 age group and gradually rises with increasing age until the 10·5 age group is reached. Diagram V shows the absolute (S. D.) and relative (C.V.) variabilities of quantity for each year of age and also the mean yields, all on the same scale of measurement. The fall in the values of the coefficient of variation with age is due mainly to the

TABLE VI.

Distributions of Quantity for each year of Age—Constants for Type IV curves fitted.

Age	No.	μ_2	μ_3	μ_4	β_1	β_2	κ_2	Mode	Mean	S.D.	Skewness	P
2·5	385	12·294	19·218	535·989	·198	3·566	·292	524·826	558·376	175·317	·191	·472
3·5	2410	8·255	7·246	225·749	·093	3·312	·207	556·660	576·410	143·660	·137	·095
4·5	1803	9·139	15·041	319·367	·296	3·823	·316	557·545	591·195	151·158	·222	·752
5·5	1554	9·896	22·182	428·741	·507	4·377	·351	602·691	644·691	157·295	·267	·909
6·5	1309	9·447	15·471	345·305	·283	3·869	·258	643·569	676·069	153·681	·211	·665
7·5	1039	10·028	14·332	378·299	·203	3·761	·176	669·084	697·184	158·341	·177	·399
8·5	913	9·779	17·479	355·001	·326	3·711	·598	671·420	710·870	156·363	·252	·428
9·5	642	10·985	20·420	457·635	·314	3·791	·398	693·504	732·554	165·723	·235	·677
10·5	510	12·199	26·833	563·754	·396	3·787	·847	688·297	737·647	174·640	·282	·711
11·5 up	674	11·527	20·511	474·104	·274	3·567	·707	683·806	723·406	169·763	·239	·922

Mean $P = ·603$

increase on mean yield with age. Cows over six years of age are relatively less variable in their yields than cows under six years of age. Cows over eleven years of age (674 in number) were studied together. The mean, mode and variability of this group are less than the corresponding values for the 10·5 age group. The curves are all positively asymmetrical and the degree of skewness runs from 0·14 to 0·28. With the exceptions of the 3·5 and 10·5 age groups, Type IV curves give good fits to the distributions of yield for the various age groups. The change in modal values with increasing age for a hundred cows of each age can be readily seen from Diagram IV, which shows graphically the percentage frequencies for each year of age.

DIAGRAM V.

Variations in Means, Standard Deviations and Coefficients of Variation of Quantity with Age of Cow.

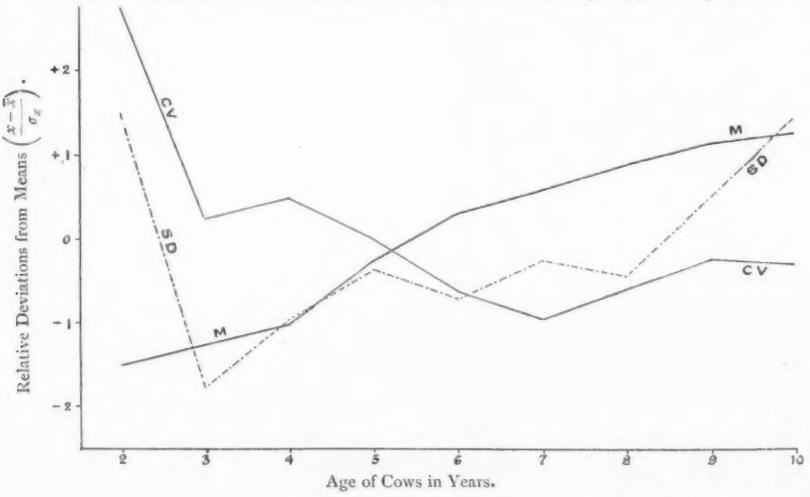


DIAGRAM VI.

1911 Data. Frequency Distribution of Percentage of Butter Fat.
Type IV Curve.

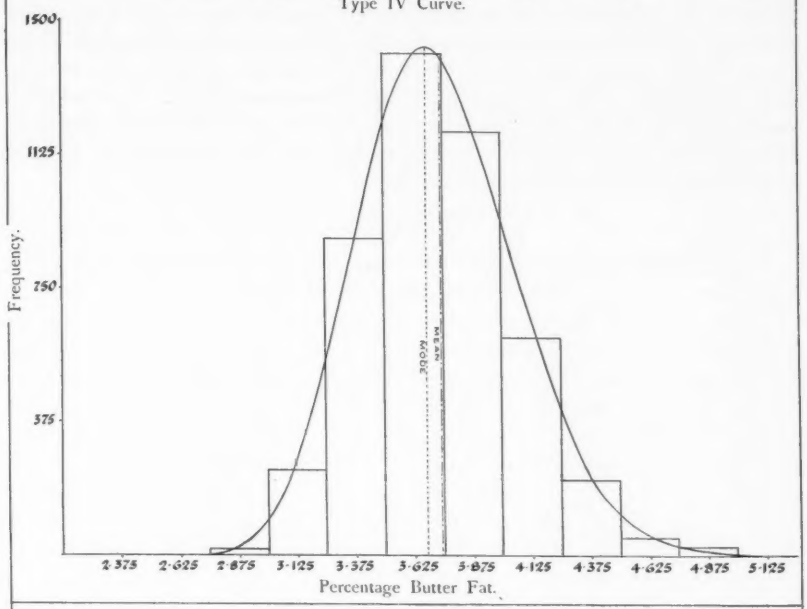
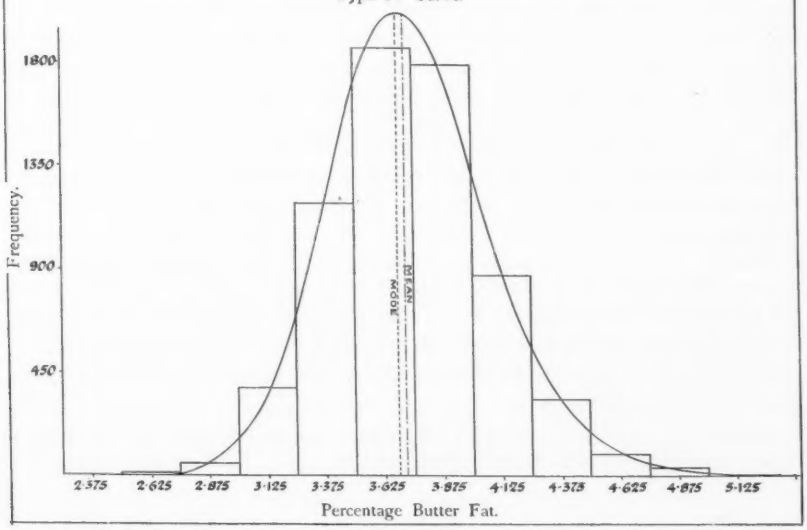


DIAGRAM VII

1912 Data. Frequency Distribution of Percentage of Butter Fat.
Type IV Curve.



(*d*) *Yield of Butter Fat for Cows all the same Age.*

A suitable unit for the study of yield of butter fat was found to be 20 pounds. As already stated, curves were fitted to yield of butter fat for cows of all ages. The material is, however, heterogeneous owing to the age factor (see Diagrams VI and VII). The theoretical and observed values for yield of butter fat of cows from three to four years of age and milked for 41 weeks were worked out. A normal curve was found to fit the distribution, 0·826 being the value of P (see Table VII). The whole problem of selection with respect to age and otherwise and its effects is discussed in a subsequent section of the paper.

TABLE VII.

Distribution of frequency of Total Butter Fat Yield of 272 *Cows aged* 3·5 *Years and milked for* 41 *Weeks.*

Total Butter Fat		Observed Frequency	Theory for Normal Curve
Sub-Range	Centre		
Up to 160	150	4	4·88
160–180	170	9	11·81
180–200	190	29	27·20
200–220	210	52	46·42
220–240	230	57	58·71
240–260	250	58	55·01
260–280	270	38	38·19
280–300	290	14	19·65
300–320	310	5	7·49
320–340	330	4	2·12
340–360	350	1	·44
360–380	370	0	·07
380–400	390	1	·01
		272	272·00

(3) *Correlations and Regressions.*

(*a*) *The Relation between Quantity and Quality of Milk.*

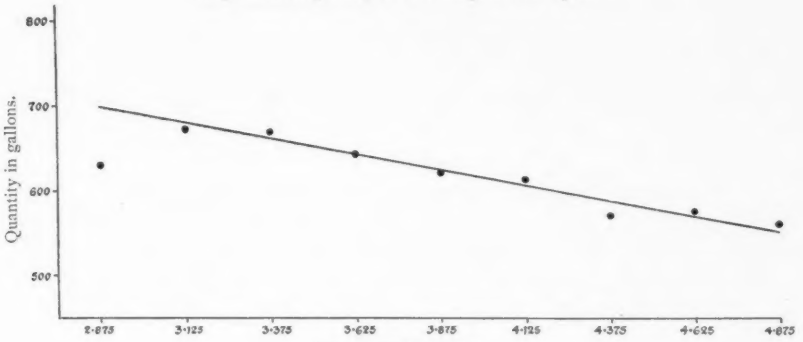
1. *Regressions.* When we consider the relationship between the quantity (Q) and quality (percentage of fat, P) we find that there is a negative correlation between these two characters which is sensible but not very great. The values of r_{PQ} are $-$·1563 and $-$·1625 for the 1911 and 1912 data respectively. This means that the greater the quantity yielded, the poorer the quality on the average*. The values of η_{PQ} are ·1673 for 1911 and ·1726 for 1912, while the values of η_{QP} are ·1639 and ·1809 for 1911 and 1912 respectively. The differences of r and the η's are not of great importance.

* I have shown elsewhere that there is a high positive correlation between *total* butter fat and quantity (see *Analyst*, Vol. LI., pp. 605, 613 (1926)).

On referring to Table IX it is seen that the divergence from linearity is somewhat greater in the 1912 data than in the 1911 data, but this is mainly accounted for by the sparse number of cows giving high and low yields. It may be safely concluded that, irrespective of age or lactation period, the regression of quantity on percentage of butter fat is practically linear between the limits of 3·3 per cent. and 4·3 per cent. of butter fat (Diagrams VIII and IX). The regression of percentage butter fat on quantity is also practically linear* (Diagrams X and XI).

DIAGRAM VIII.

Regression of Quantity on Percentage of Fat. 1911.



2. *Scedastic Curves.* If we confine our attention to particular values of yield of milk for a lactation period, we can put this question: Is there great variability in percentage of butter fat among cows giving a particularly high or a particularly low yield of milk? The values of the variance for the fat percentage arrays of cows giving a particular yield would be a preliminary test. The distribution of the variance of the percentage arrays for sub-ranges of quantity was found on analysis to be linear and heteroscedastic (Diagram XII). Since the standard deviation is used as a measure of variability, diagrams of scedastic curves have been drawn in order to show how the standard deviations of the arrays themselves vary with increasing yield or increasing percentage (Diagram XIII). It will be seen that with a low yield for a lactation period—say 300 gallons—the variability of the percentage array is comparatively high (about 0·4 per cent.), while the corresponding variability for a high yield—say 900 gallons—is about 0·3 per cent. Thus heavy milkers, that is, those giving a high total yield for a lactation period, show less variation in the butter fat content than low yielding cows. A group of bad cows is, therefore, a group giving low yields and with great variation among themselves as regards the percentage of butter fat.

If we look at the converse problem, it is found that cows giving a low percentage of butter fat are very variable in milk yield. For 1911, it is seen that cows giving 3 per cent. of butter fat have a standard deviation of about 170 gallons in their

DIAGRAM IX.

Regression of Quantity on Percentage of Fat. 1912.

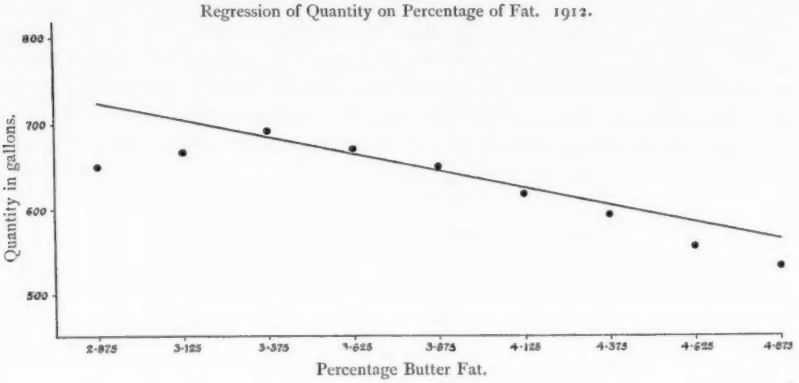


DIAGRAM X.

Regression of Percentage of Fat on Quantity. 1911.

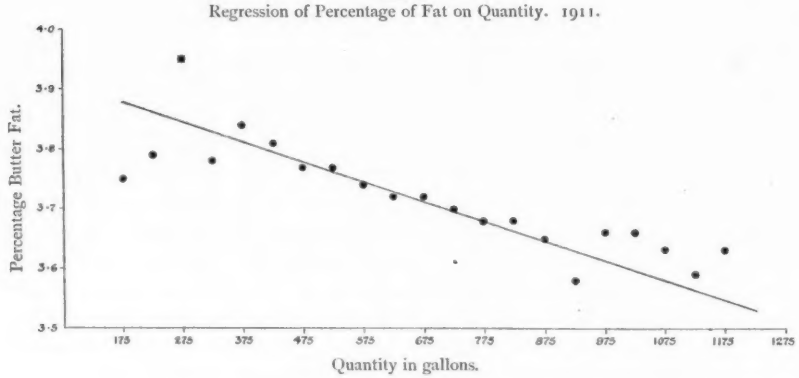


DIAGRAM XI.

Regression of Percentage of Fat on Quantity. 1912.

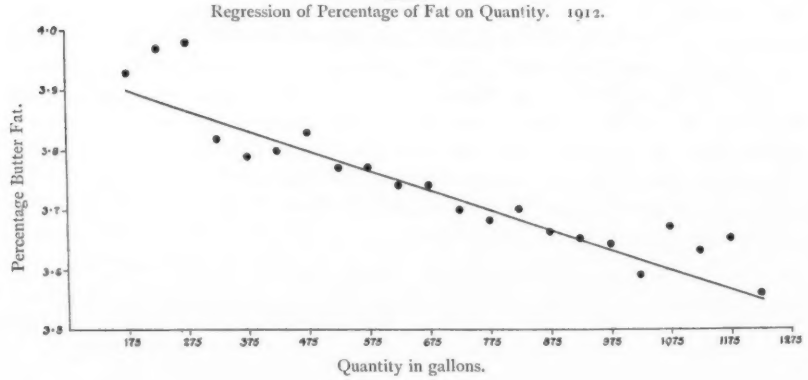


DIAGRAM XII.

Regression of Q Variance on P (I and II).
Regression of P Variance on Q (III and IV).

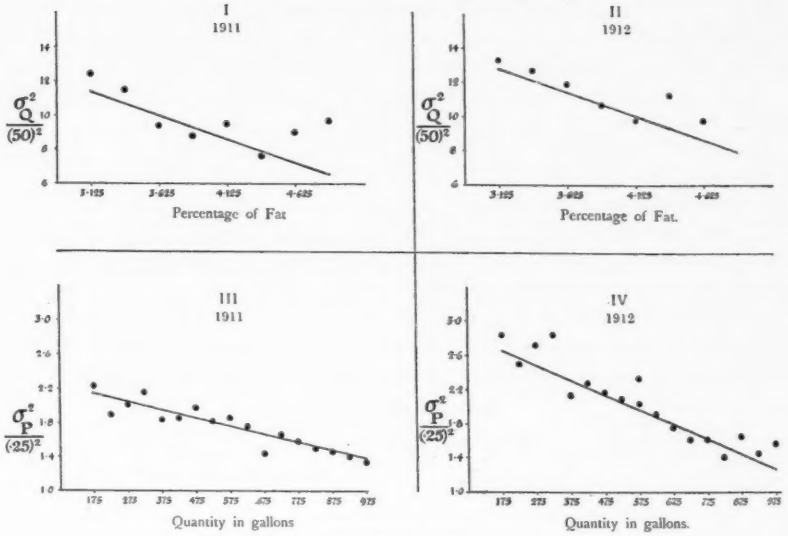
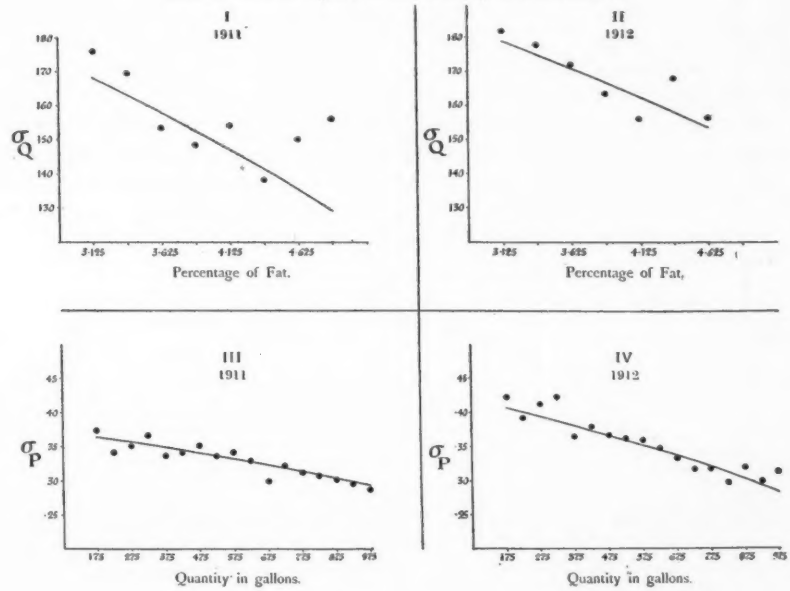


DIAGRAM XIII.

Scedastic Curves for σ_Q on P (I and II) and σ_P on Q (III and IV).



milk yield, while those yielding about 4·5 per cent. of butter fat have a variability of about 140 gallons. The corresponding figures for 1912 are 180 and 150 approximately.

(*b*) *Quantity and Age.*

Regression of Quantity on Age.

The values of the correlation coefficient r_{AQ} are ·3216 for 1911 and ·3418 for 1912. It is thus evident that there is a considerable relationship between age and quantity. The corresponding values of η_{QA} are ·3687 and ·3656. The divergence between η and r for each year is probably significant (Table IX). The regression, therefore, is not linear. A regression parabola was accordingly fitted to the data for each year (Diagrams XIV and XV). These parabolae would appear to fit the data with a considerable degree of accuracy, at least for the younger and more important ages. The maximum average yield of milk, therefore, occurs amongst cows of $13\frac{1}{2}$ years of age in the 1911 data, and amongst cows of $12\frac{1}{2}$ years of age in the 1912 data, the difference in the ages giving the maximum yield being thus about one year. If we concentrate our attention on Diagrams XIV and XV and the relevant tables, we can

DIAGRAM XIV.

Regression of Quantity on Age. 1911.

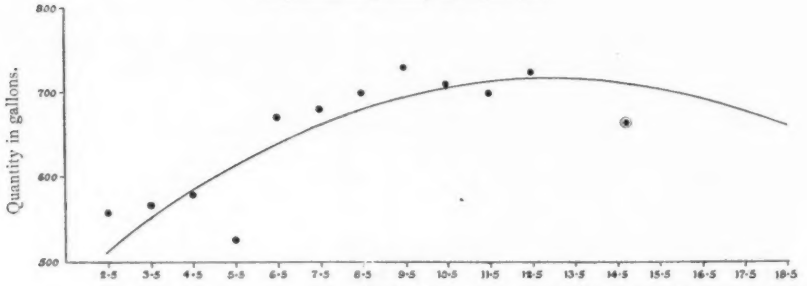
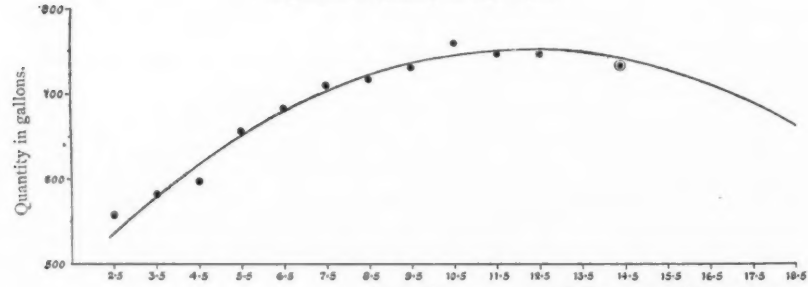


DIAGRAM XV.

Regression of Quantity on Age. 1912.



extract information useful to the dairy farmer. The dairy farmer might put this question. I have a herd of x cows of various ages. Can you give me any idea from the known results regarding quantity and age, what total yield of milk I might expect to get during a lactation period from my herd of x cows? The answer is in the affirmative. Suppose a farmer had a herd of 10 cows whose ages are those given in the following table:

TABLE VIII.

No. of cows	Ages of cows	Yield per cow for lactation period	Total yield of cows for lactation period
2	4	617	1234
1	5	651	651
1	6	680	680
1	7	704	704
2	9	737	1474
2	11	752	1504
1	13	746	746

Total yield in gallons for above 10 cows = 6993.
Mean yield per cow for a lactation period of 37·41 weeks = 699·3.

This table shows the total yields of milk to be expected at the various ages and the most probable total yield of milk from the 10 cows. This number is 6993 gallons, or an average of 699·3 gallons per cow, for an average lactation period of 37·41 weeks.

(*c*) *Fat Percentage and Age.*

Regression of Fat Percentage on Age.

The values of the correlation coefficient for 1911 and 1912 are $r_{PA} = -\cdot2275$ and $-\cdot2336$ respectively, which would indicate that the average fat percentage decreased with age. The values of η_{PA} for 1911 and 1912 are ·2618 and ·2693 respectively. The results of the tests for linearity are given in Table IX, where the values of $(\eta - r)$ are compared with their probable errors. The results show that the regression curves of percentage on age are non-linear for both years. Regression parabolae were fitted to the data and the results are shown in Diagrams XVI and XVII. On

TABLE IX.

Table of Correlation Coefficients.

Characters	1911 Data r	1911 Data E_r	1912 Data r	1912 Data E_r
Age and Quantity	+·3216	±·0089	+·3418	±·0073
Age and Weeks	−·0827	±·0099	−·0379	±·0083
Age and Fat Percentage	−·2275	±·0094	−·2336	±·0078
Age and Yield of Butter Fat	+·2381	±·0092	+·2650	±·0077
Fat Percentage and Quantity	−·1563	±·0096	−·1625	±·0081
Quantity and Weeks	+·5734	±·0067	+·6139	±·0052
Percentage and Weeks	+·0777	±·0099	+·0540	±·0082

TABLE IX (*cont.*)

Test for Linearity of Regression.

Characters	1911 Data					1912 Data				
	N	η	r	$(\eta - r)$	$\frac{(\eta - r)}{E(\eta - r)}$	N	η	r	$(\eta - r)$	$\frac{(\eta - r)}{E(\eta - r)}$
Quantity on Age	4591	·3687	+ ·3216	·0471	9·05	6648	·3656	+ ·3418	·0238	7·84
Fat Percentage on Age	4591	·2618	− ·2275	·0343	6·50	6648	·2693	− ·2336	·0357	8·10
Yield of Butter Fat on Age	4591	·2855	+ ·2381	·0474	7·92	6648	·2858	+ ·2650	·0208	6·46
Weeks in Milk on Age	4557	·1601	− ·0827	·0774	6·86	6648	·1359	− ·0379	·0980	7·89
Quantity on Fat Percentage	4671	·1639	− ·1563	·0076	2·50	6648	·1809	− ·1625	·0184	4·80
Fat Percentage on Quantity	4671	·1673	− ·1563	·0110	3·02	6648	·1726	− ·1625	·0101	3·51
Quantity on Weeks in Milk	4591	·5789	+ ·5734	·0055	4·00	6648	·6205	+ ·6139	·0066	5·45
Weeks in Milk on Quantity	4591	·5912	+ ·5734	·0178	7·23	6648	·6300	+ ·6139	·0161	8·55
Weeks in Milk on Fat Percentage	4557	·0861	+ ·0777	·0084	1·85	6648	·0763	+ ·0540	·0223	3·20
Fat Percentage on Weeks in Milk	4557	·1192	+ ·0777	·0415	4·53	6648	·0990	+ ·0540	·0450	5·01

DIAGRAM XVI.

Regression of Percentage Butter Fat on Age. 1911.

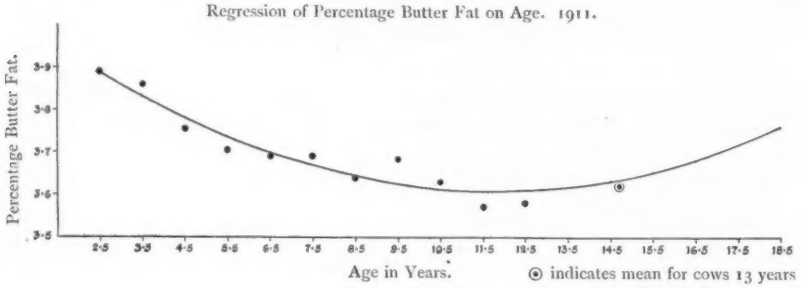
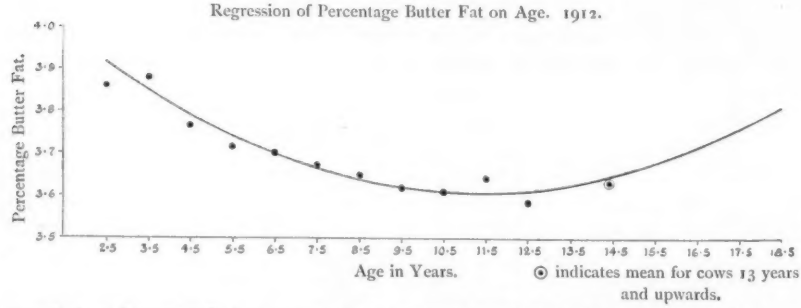


DIAGRAM XVII.

Regression of Percentage Butter Fat on Age. 1912.



examining these parabolae, we see that they are practically identical. In both cases the percentage of fat decreases with age until it reaches a minimum value of slightly above 3·6 at 11 years of age, after which the percentage appears to increase with increasing age.

(*d*) *Quantity and Duration of Lactation Period.*

(1) Regression of quantity on weeks in milk.

The correlation between the length of the lactation period and the total quantity of milk is, as was to be expected, very considerable. The values for the correlation coefficient for 1911 and 1912, respectively, are ·5734 and ·6139, while the corresponding values of η_{QW} are ·5789 and ·6205. The results in Table IX show that, for each year, the regression deviates slightly from linearity. The deviation for 1912 is found to be greater than the corresponding deviation for 1911. The results are shown in Diagrams XVIII and XIX.

DIAGRAM XVIII.

Regression of Quantity on Length of Lactation Period. 1911.

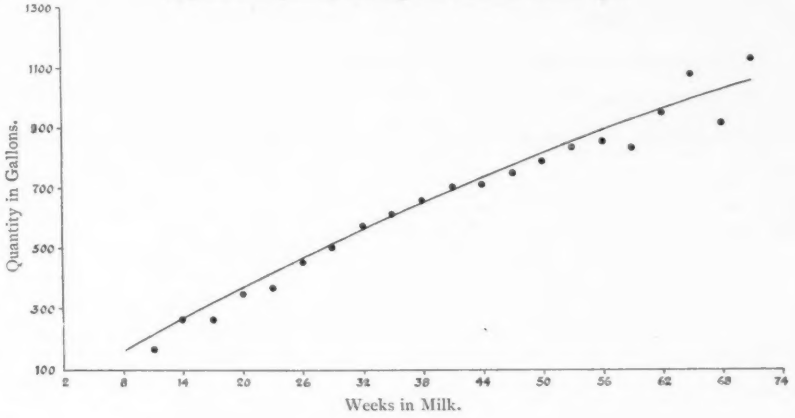
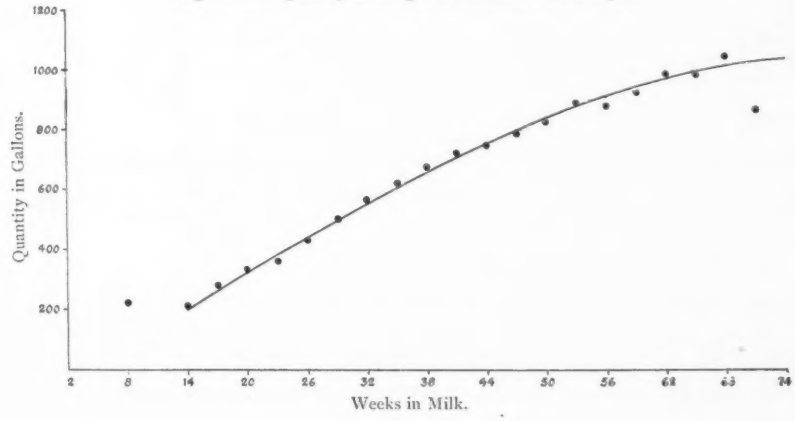


DIAGRAM XIX.

Regression of Quantity on Length of Lactation Period. 1912.



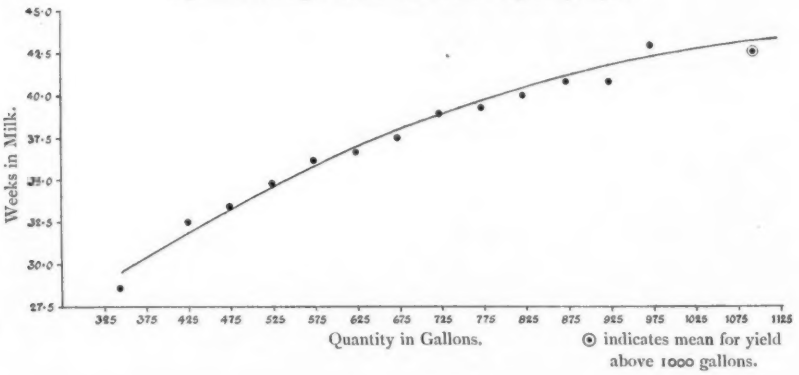
If we exclude from the data all cows whose lactation periods exceed 56 weeks, because lactation periods exceeding 56 weeks are unusual, we get slightly different values of η and r. The value of r_{QW} for all cows of lactation periods less than 56 weeks is found to be ·5435, while the values of η_{QW} and η_{WQ} are ·5487 and ·5614 respectively. These results are for 1911. The regression of quantity on lactation period (with maximum period of 56 weeks) also deviates from linearity. A close examination of the results shows that for the most important periods, namely, between 26 and 56 weeks, the regression of average quantity on length of lactation period is linear. The departure from linearity in the case of very high and very low lactation periods is mainly due to the fact that the very long lactation periods are abnormal and the very short lactation periods are incomplete, due to the manner of making the records by the Association. In the study of the 1920 records these abnormal lactation periods were excluded.

(2) Regression of weeks in milk on quantity.

The values of η_{WQ} for 1911 and 1912, respectively, are ·5912 and ·6300, while the corresponding values of the ratio $\frac{(\eta - r)}{E_{(\eta - r)}}$ are 7·23 and 8·55. Even excluding cows with lactation periods greater than 56 weeks this ratio is not reduced. Thus it is clear that the relationship between quantity and average weeks in milk is non-linear. The regressions are shown graphically on Diagrams XX and XXI.

DIAGRAM XX.

Regression of Length of Lactation Period on Quantity. 1911.



(e) *Fat Percentage and Length of Lactation Period.*

(1) Regression of percentage of butter fat on weeks in milk.

The correlation between the average percentage of butter fat and the number of weeks in milk is small though distinctly positive. The values of the correlation coefficients for 1911 and 1912 are + ·0777 and + ·0540, values which cannot be

DIAGRAM XXI.

Regression of Length of Lactation Period on Quantity. 1912.

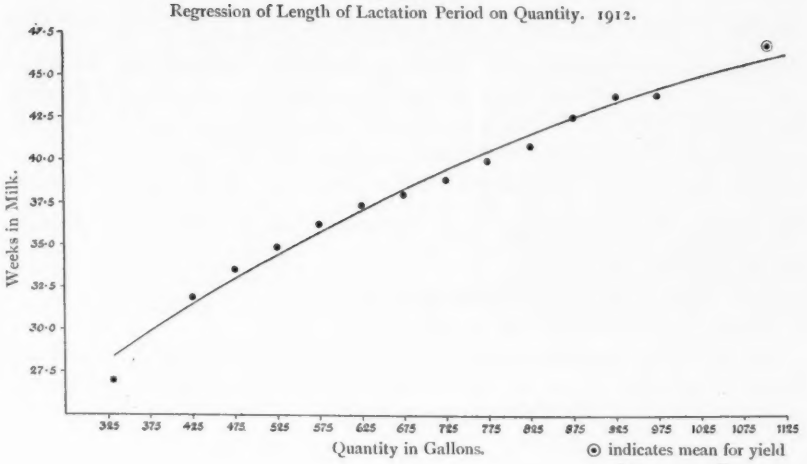


DIAGRAM XXII.

Regression of Percentage Butter Fat on Length of Lactation Period. 1911.

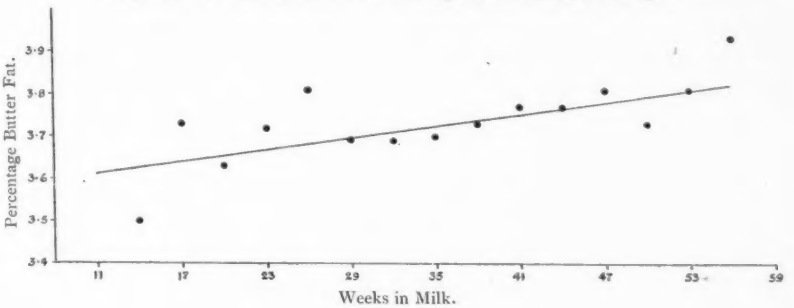
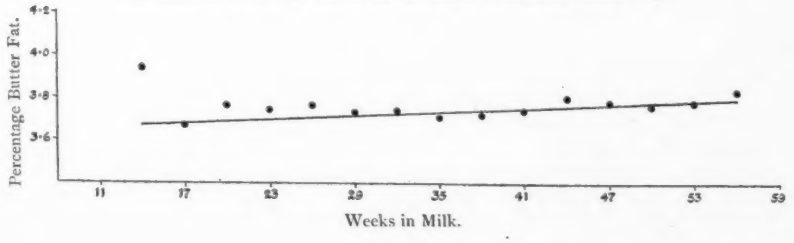


DIAGRAM XXIII.

Regression of Percentage Butter Fat on Length of Lactation Period. 1912.



regarded as being significantly different. The corresponding values of η_{PW} are ·1192 and ·0990, while the values of $\frac{(\eta - r)}{E_{(\eta - r)}}$ are 4·53 and 5·01 for 1911 and 1912 respectively. The regression therefore is distinctly non-linear, though the deviations from linearity occur mainly for very high and very low lactation periods where the frequencies are few and unimportant practically (Diagrams XXII and XXIII).

(2) Regression of weeks in milk on percentage of butter fat.

The values of η_{WP} for 1911 and 1912 are ·0861 and ·0763 respectively, while the corresponding values of $\frac{(\eta - r)}{E_{(\eta - r)}}$ are 1·85 and 3·20, showing that the regressions are practically linear.

(*f*) *Age and Length of Lactation Period.*

Regression of weeks in milk on age.

The correlation between age and average number of weeks in milk is small and negative, the values of the correlation coefficient for 1911 and 1912, respectively,

DIAGRAM XXIV.

Regression of Length of Lactation Period on Age. 1911.

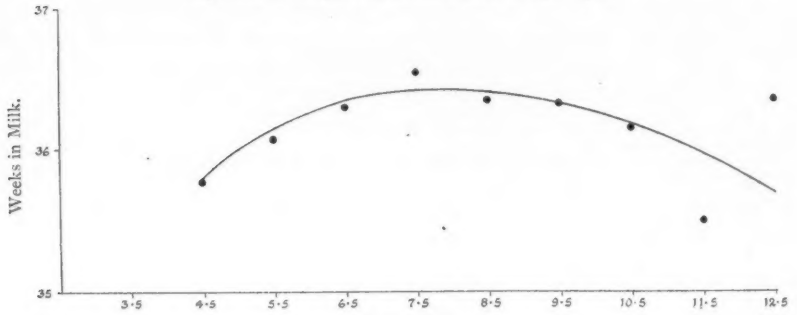
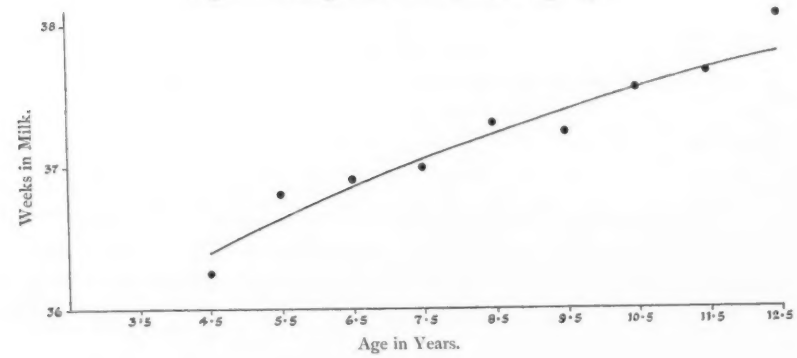


DIAGRAM XXV.

Regression of Length of Lactation Period on Age. 1912.



being $-\cdot0827$ and $-\cdot0379$. The corresponding values of η_{WA} are $\cdot1601$ and $\cdot1359$ respectively. The regressions are non-linear, but this may be due to the nature of the selected sample rather than to a real departure from linearity (Diagrams XXIV and XXV, see also Table XXXI of Appendix). It was found that heifers and young cows under 4 years of age had significantly shorter lactation periods than older cows for both years 1911 and 1912. For the year 1911, older cows show an increase in lactation period with age until 8 years is reached and thereafter there is a decrease. For the year 1912, there is an increase in lactation period with age from 4 years to 13 years. See Diagrams XXIV and XXV.

The rise in average lactation period in 1912 as compared with 1911 is of practical interest. The data for succeeding years would reveal whether or not there is a secular change in progress of duration of lactation period with age.

(*g*) *Age and Total Yield of Butter Fat.*

Regression of butter fat yield on age.

The values of the correlation coefficients r_{AB} are $+\cdot2381$ and $+\cdot2650$ for 1911 and 1912 respectively, showing a fairly high correlation between these two factors. The corresponding values of η_{BA} are $\cdot2855$ and $\cdot2858$, while the values of $\frac{(\eta - r)}{E_{(\eta - r)}}$ are 7·92 and 6·46 respectively, showing that there is a distinct deviation from linearity. Regression parabolae were fitted to the data for both years (Diagrams XXVI and XXVII). There are higher yields of butter in 1912 than in 1911, due mainly to longer lactation periods in 1912 when compared with 1911 (see previous paragraph).

DIAGRAM XXVI.

Regression of Yield of Butter Fat on Age. 1911.

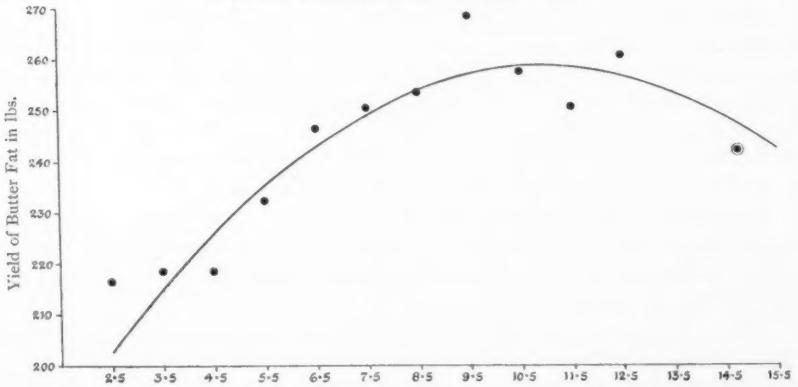
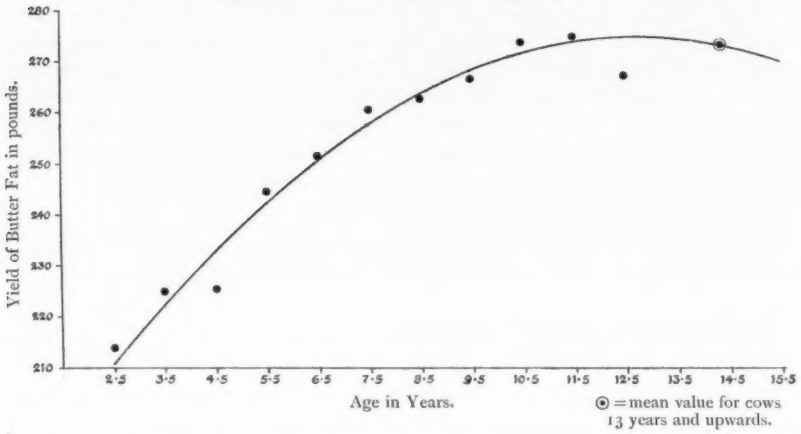


DIAGRAM XXVII.

Regression of Yield of Butter Fat on Age. 1912.



(h) The Effect of Age and Length of Lactation Period on the Correlation between Quantity and Quality.

As a preliminary study, a number of partial correlation coefficients have been calculated in order to see, for example, what the correlation would be between Percentage and Quantity for constant Age. These partial correlation coefficients however are based on the assumption that the regressions in all case are linear. This is true in a few cases, and in a number of cases the regressions are approximately linear. In the following discussion the figures given are therefore such as would occur if linearity held throughout. In a future paper the correlations will be given corrected for age by referring to a standard age.

When the correlation coefficient between quantity and quality for a constant lactation period is calculated, it is found that its absolute value is greater than the general value. For 1911 (see Tables IX and X) we have $r_{PQ.W} = -\cdot2459$ compared with $r_{PQ} = -\cdot1563$, and for 1912 the corresponding figures are $-\cdot2483$ and $-\cdot1625$ respectively. This means that if we eliminate the effect of the length of lactation period, there is, on the average, a greater fall in quality as the quantity rises than is the case irrespective of the duration of the lactation period.

If we consider the value of the correlation coefficient for a constant age, we find that it falls from $r_{PQ} = -\cdot1563$ to $r_{PQ.A} = -\cdot0901$. That is to say, there is a reduction in the relationship but the correlation is still significant. For 1912 the values are $r_{PQ} = -\cdot1625$ and $r_{PQ.A} = -\cdot0905$.

If we take into account both age and length of lactation period, that is, if we eliminate both these factors, we have again a negative correlation, the value being $r_{PQ.AW} = -\cdot1594$ as against $r_{PQ} = -\cdot1563$ for 1911, and $r_{PQ.AW} = -\cdot1629$ compared with $r_{PQ} = -\cdot1625$ for 1912.

TABLE X.

Partial Correlation Coefficients.

Complete Data.

Characters	1911	1912
$r_{PQ.A}$	−·0901	−·0905
$r_{QW.A}$	+·6042	+·6675
$r_{PW.A}$	+·0607	+·0465
$r_{AQ.P}$	+·2974	+·3167
$r_{AQ.W}$	+·4382	+·4628
$r_{\quad .W}$	−·2459	−·2483
$r_{AP.Q}$	−·1895	−·1920
$r_{AP.W}$	−·2278	−·2320
$r_{QW.P}$	+·5642	+·6319
$r_{AP.Q}$	−·3239	−·3340
$r_{AW.P}$	−·0732	−·0262
$r_{PQ.AW}$	−·1594	−·1629
$r_{AQ.PW}$	+·4113	+·4302
$r_{AP.QW}$	−·1358	−·1364
	Abridged Tables.	
r_{PQ}	−·2372	—
$r_{QW.P}$	+·5947	—
	Curtailed Tables.	
$r_{PQ.A}$	−·1159	−·1205
$r_{AQ.P}$	+·1721	+·2244
$r_{AQ.W}$	+·2940	+·3505
$r_{AP.Q}$	−·2027	−·2016
$r_{AW.Q}$	−·2335	−·2532
$r_{PQ.AW}$	−·1843	−·1833
$r_{AP.QW}$	−·1726	−·1598
	Complete Data.	
$r_{(PQ).A}$*	+·2385	+·2591
$r_{(PQ).W}$	+·5965	+·6299
$r_{(PQ)A.W}$	+·3599	+·3646
$r_{(PQ)W.A}$	+·6367	+·6628

Abridged means no weeks above the 56 class taken in W and Q table.
Curtailed means Q from 701 gallons.

* (PQ) means $P \times Q$.

The true expression of the relationship between quantity and quality is that which gives it for constant conditions, that is to say, $r_{PQ.AW}$ expresses accurately that relationship. The results show that, if we consider the relationship between

P and Q for constant age only, the relationship between these two factors is considerably decreased ($-\cdot1594$ to $-\cdot0901$) and this decrease may therefore be ascribed to the operation of a variable lactation period. On the other hand, if we consider the PQ relationship for a constant lactation period *and* without considering age, the relationship increases (when compared with $r_{PQ.\,AW} = -\cdot1594$) to $-\cdot2459$. That is to say, the effect of varying age with a constant lactation period is to increase the correlation between P and Q when compared with the corresponding correlation for a constant age and a constant lactation period. It is therefore natural to expect that, if the PQ relationship is considered without reference to age or lactation period—both these factors varying—the correlation found should be intermediate between $r_{PQ.\,A} = -\cdot0901$ and $r_{PQ.\,W} = \cdot2459$, namely, $r_{PQ} = -\cdot1563$, which closely approximates $r_{PQ.\,AW} = \cdot1594$.

The reduction from $r_{PQ.\,AW} = -\cdot1594$ to $r_{PQ.\,A} = -\cdot0901$ is seen to be due to the fact that there is a high correlation between W and Q (see page 152 and Table XX) and an insignificant correlation between W and P. Quantity is affected by length of lactation period while butter fat percentage is not. The net effect is to make the relationship between P and Q more of a random character when all lactation periods are considered for a constant age.

The increase from $r_{PQ.\,AW} = -\cdot1594$ to $r_{PQ.\,W} = -\cdot2459$ is seen to be due to the effects of age on butter fat percentage and upon quantity. Since the correlations $r_{PA.\,W}$ and $r_{QA.\,W}$ are high, it is natural to expect a less random relationship between P and Q for a constant lactation period, irrespective of age, when compared with the relationship $r_{PQ.\,AW}$, the correlation for a constant lactation period and a constant age.

(4) *The average total Butter Fat Yield associated with a Definite Age and a Definite Length of Lactation Period.*

A problem of practical importance is the determination of the average quantity of butter fat obtained from cows of various ages and for varying lactation periods. The regression equations between age and other factors clearly show that the regressions are non-linear and parabolic regressions. On applying the test for linearity (Pearson, *Skew Correlation and Non-linear Regression*, Biometric Series, I. pp. 1—54, Drapers' Research Memoir) the following results were obtained:

TABLE XI.

	$\eta^2 - r^2$	$\phi_2(\eta^2 - r^2) - \epsilon^2$
Age and Yield of Butter Fat	·011461	·007338
Age and Percentage	·017962	·010040
Age and Quantity	·016837	·006377

It is clear from the above table that in each case a parabola would give a better fit than a straight line. Since the regressions are non-linear, an attempt was made

to find a multiple regression surface of the second degree in accordance with the method described by Rae in Appendix XI of the *Report on Twelfth Decennial Census of Scotland*, 1911, Vol. III. The results were unsatisfactory and the problem was then attacked in the following manner: Two sets of regression parabolae were calculated, the first set giving the mean yield of butter fat for any age of cow with various constant lactation periods, and the second showing the mean yield of butter fat for any length of lactation period with various constant ages. In this way two values of B_{AW} (= yield of butter fat for a lactation period) were obtained for each combination of values of A and W, and although the agreement between these values is not completely satisfactory the differences are small throughout a large part of the surface.

The means of the two values for each cell were now calculated and it is clear that these would not form a regular surface. The question may be put: Why are we not content with this new series of means?

The answer is twofold in character. Firstly, we want a single formula to predict yield of butter fat from age and length of lactation period for any given age and any given number of weeks in milk. Secondly, even as they stand, the values of B_{AW} found from the two series of regression parabolae will vary according to the actual numbers observed in the arrays of each series. For instance, it was found that the regression parabola for the class 50 weeks in milk diverged from the general run of parabolae for "weeks" classes, because of the small numbers of cases observed compared with the number of cases in the other arrays. It must be borne in mind that each regression parabola has been found independently of its neighbour, and we have as yet found no general equation based on the whole series of regression parabolae of ages for "weeks" classes or vice versa, or the means of these two series. We can however consider the problem of fitting a surface to these means by the method of moments which Pearson has shown (*Biometrika*, Vol. II. p. 12 *et seq.*) to give theoretically good fits in ordinary curve fitting. As in the case of curves, so also with surfaces, goodness of fit depends more on the nature of the equation to the surface selected, than upon the number of constants for the surface. A model of the surface was prepared and from the nature of the surface obtained it seemed reasonable to suppose that a general equation of the second degree would give a good fit.

For various values of (say) x and y we are given several values of (say) Z centering round the mean value. It is required to find a function $f(xy)$ so that given a value of x and of y, $z = f(xy)$ will adequately represent the most probable value of Z. We shall accordingly assume that $f(x, y)$ is a polynomial of the second degree, i.e.

$$Z = Z_0(a + b_1x + b_2y + c_{20}x^2 + c_{11}xy + c_{02}y^2),$$

where Z_0, a, b_1, etc. are constants to be determined from the data by the method of moments.

To obtain the necessary equations we follow Pearson (*Biometrika*, Vol. II. p. 12).

Let

$$Z = Z_0\left\{a + b_1\frac{x}{l} + b_2\frac{y}{k} + c_{20}\left(\frac{x}{l}\right)^2 + c_{11}\left(\frac{x}{l}\right)\left(\frac{y}{k}\right) + c_{02}\left(\frac{y}{k}\right)^2 + d_{30}\left(\frac{x}{l}\right)^3 + \ldots + e_{40}\left(\frac{x}{l}\right)^4 + \ldots\right\},$$

the origin being at mid x array and the mid y array, $2l$ the x range and $2k$ the y range.

We shall put $Z_0 = \dfrac{M_{00}}{2k\,2l}$ = the mean observed. Multiply by $\left(\dfrac{x}{l}\right)^{2p}\left(\dfrac{y}{k}\right)^{2q}$ and equate the $(2p, 2q)$th moment of the observed Z to the integral

$$l^{2p}k^{2q}\int_{-k}^{+k}\int_{-l}^{+l} Z\left(\frac{x}{l}\right)^{2p}\left(\frac{y}{k}\right)^{2q}dx\,dy,$$

which is the $(2p, 2q)$th moment of the theoretical Z. Thus we obtain

$$\frac{M_{2p,2q}}{l^{2p}k^{2q}} = Z_0\,2l\,2k\left\{\frac{a}{(2p+1)(2q+1)} + \frac{c_{20}}{(2p+3)(2q+1)} + \frac{c_{02}}{(2p+1)(2q+3)} + \frac{l_{40}}{(2p+5)(2q+1)} + \frac{l_{22}}{(2p+3)(2q+3)} + \ldots\right\}.$$

Let $$\lambda_{2p,2q} = \frac{M_{2p,2q}}{M_{00}l^{2p}k^{2q}}.$$

Therefore

$$\lambda_{2p,2q} = \frac{a}{(2p+1)(2q+1)} + \frac{c_{20}}{(2p+3)(2q+1)} + \frac{c_{02}}{(2p+1)(2q+3)} + \ldots.$$

If we multiply the equation by $\left(\dfrac{x}{l}\right)^{2p+1}\left(\dfrac{y}{k}\right)^{2q}$ as above we have

$$\frac{M_{2p+1,2q}}{l^{2p+1}k^{2q}} = Z_0\,2l\,2k\left\{\frac{b_1}{(2p+3)(2q+1)} + \frac{d_{30}}{(2p+5)(2q+1)} + \frac{d_{12}}{(2p+3)(2q+3)} + \ldots\right\},$$

or $$\lambda_{2p+1,2q} = \frac{b_1}{(2p+3)(2q+1)} + \frac{d_{30}}{(2p+5)(2q+1)} + \ldots.$$

Similarly

$$\lambda_{2p,2q+1} = \frac{b_2}{(2p+1)(2q+3)} + \frac{d_{21}}{(2p+3)(2q+3)} + \frac{d_{03}}{(2p+1)(2q+5)} + \ldots,$$

and $$\lambda_{2p+1,2q+1} = \frac{c_{11}}{(2p+3)(2q+3)} + \frac{e_{31}}{(2p+5)(2q+3)} + \frac{e_{13}}{(2p+3)(2q+5)} + \ldots.$$

The λ's are known and these are the equations to find the constants. Thus in the particular case where only the terms of the second degree are retained

$$(1)\quad \lambda_{00} = a + \frac{c_{20}}{3.1} + \frac{c_{02}}{1.3},$$

$$(2)\quad \lambda_{01} = \frac{b_2}{1.3},$$

$$(3)\quad \lambda_{10} = \frac{b_1}{3.1},$$

$$(4)\quad \lambda_{11} = \frac{c_{11}}{3.3},$$

$$(5)\quad \lambda_{20} = \frac{a}{3.1} + \frac{c_{20}}{5.1} + \frac{c_{02}}{3.3},$$

$$(6)\quad \lambda_{02} = \frac{a}{1.3} + \frac{c_{20}}{3.3} + \frac{c_{02}}{1.5}.$$

These are six equations to determine the six constants from which we have

$$a = \tfrac{7}{2}\lambda_{00} - \tfrac{15}{4}(\lambda_{20} + \lambda_{02}),$$
$$b_1 = 3\lambda_{10},$$
$$b_2 = 3\lambda_{01},$$
$$c_{11} = 9\lambda_{11},$$
$$c_{20} = \tfrac{15}{4}(3\lambda_{20} - \lambda_{00}),$$
$$c_{02} = \tfrac{15}{4}(3\lambda_{02} - \lambda_{00}).$$

In applying the above to the means obtained from the two sets of parabolae the ranges selected as being reliable in involving large numbers of observations were 2 years to 11 years inclusive for age, giving $2l = 9$ and 26 weeks to 50 weeks inclusive, for lactation period giving $2k = 8$.

In taking the means between the two sets of parabolae we obtain a series of points, and when adjacent points are connected by straight lines we have a trapezoidal solid which contains a number of systems of trapezia parallel to the xz plane and also a number of systems parallel to the yz plane. Accordingly we require to find corrective terms for the moments of the solid by Pearson's method as given for moments of a system of trapezia (*Biometrika*, Vol. II. p. 8).

Suppose P, Q, R, S to be the tops of ordinates (our mean points) at $x_r y_s$, $x_{r+1} y_s$, $x_{r+1} y_{s+1}$, $x_r y_{s+1}$. Let a straight line meeting PS and QR always parallel to the xz plane move from PQ to SR. In so moving it will generate a hyperbolic paraboloid and we obtain the same solid whether we take the moving line parallel to the XZ plane and meeting PS and QR or parallel to the YZ plane meeting PQ and SR. It is clear that the inclination of the moving lines to the xy plane varies continuously. The following are the necessary corrective terms (see *Biometrika*, Vol. II. p. 9), the notation being adapted where necessary to our special case:

$${}_aM_0 = {}_aM_0' - L_0(y_m + y_0),$$
$${}_aM_1 = {}_aM_1' - L_1(y_m - y_0).$$

${}_aM_r$ is the rth moment of the trapezoidal area with constant age a.

Similarly,

$${}_wM_0 = {}_wM_0' - L_0(y_m + y_0),$$
$${}_wM_1 = {}_wM_1' - L_1(y_m - y_0),$$
$$M_{00} = S({}_wM_0) - L_0({}_wy_m + {}_wy_0)$$
$$= S({}_aM_0) - L_0({}_ay_m + {}_ay_0),$$
$$M_{01} = S({}_wM_1) - L_1({}_wy_m - {}_wy_0),$$
$$M_{10} = S({}_aM_1) - L_1({}_ay_m - {}_ay_0),$$
$$M_{11} = S\{({}_wM_1) \times a\} - L_1({}_wy_m - {}_wy_0)$$
$$= S\{({}_aM_1) \times w\} - L_1({}_ay_m - {}_ay_0),$$
$$M_{02} = M'_{02} - L_2({}_wy_m + {}_wy_0),$$
$$M_{20} = M'_{20} - L_2({}_ay_m + {}_ay_0),$$

where
$$M'_{02} = S\{{}_aM_0 \times w^2\} + \frac{S({}_aM_0)}{6},$$

and
$$M'_{20} = S\{{}_wM_0 \times a^2\} + \frac{S({}_wM_0)}{6}.$$

The values obtained from the data are

$$\lambda_{00} = 1 \quad , \qquad \lambda_{20} = \cdot 3255465,$$
$$\lambda_{10} = \cdot 0439397, \qquad \lambda_{11} = \cdot 0024236,$$
$$\lambda_{01} = \cdot 1006218, \qquad \lambda_{02} = \cdot 3310849,$$
$$z_0 = 257 \cdot 4375.$$

Hence we have the following values of the constants

$$a = 1 \cdot 0376323, \qquad c_{20} = - \cdot 0876019,$$
$$b_1 = \cdot 1318191, \qquad c_{11} = \cdot 0218124,$$
$$b_2 = \cdot 3018654, \qquad c_{02} = - \cdot 0252929,$$

and the required equation is

$$Z = 267 \cdot 1254652 + 33 \cdot 9351795 \frac{x}{l} + 77 \cdot 7114739 \frac{y}{k} - 22 \cdot 5520141 \frac{x^2}{l^2} + 5 \cdot 6153297 \frac{xy}{lk} - 6 \cdot 5118558 \frac{y^2}{k^2},$$

or, since $l = 4 \cdot 5$ and $k = 4$,

$$= 267 \cdot 1254652 + 7 \cdot 5411510x + 19 \cdot 4278684y - 1 \cdot 1136797x^2 + \cdot 3119628xy - \cdot 4069910y^2.$$

The following Table (Table XII) shows (1) the means of values from the regression parabolae (Roman figures), (2) the values obtained from the above equation (Italic figures), and (3) the observed values (Heavy type). It is seen that the equation fairly describes the observations.

The equation shows a range of yields of butter fat from 131 pounds for a cow two years old milking for 26 weeks to 373 pounds for a cow aged 11 years milked for 53 weeks. If we calculate this to butter containing the maximum amount of water legally permissible, viz. 16 per cent., we have a range from $\frac{131 \times 100}{84} = 156$ pounds to $\frac{373 \times 100}{84} = 444$ pounds. A reproduction is shown of the surface on Plate II (Diagram XXVIII). The number of cows for each year of age, and weeks in milk are given in Table LXIX of the Appendix.

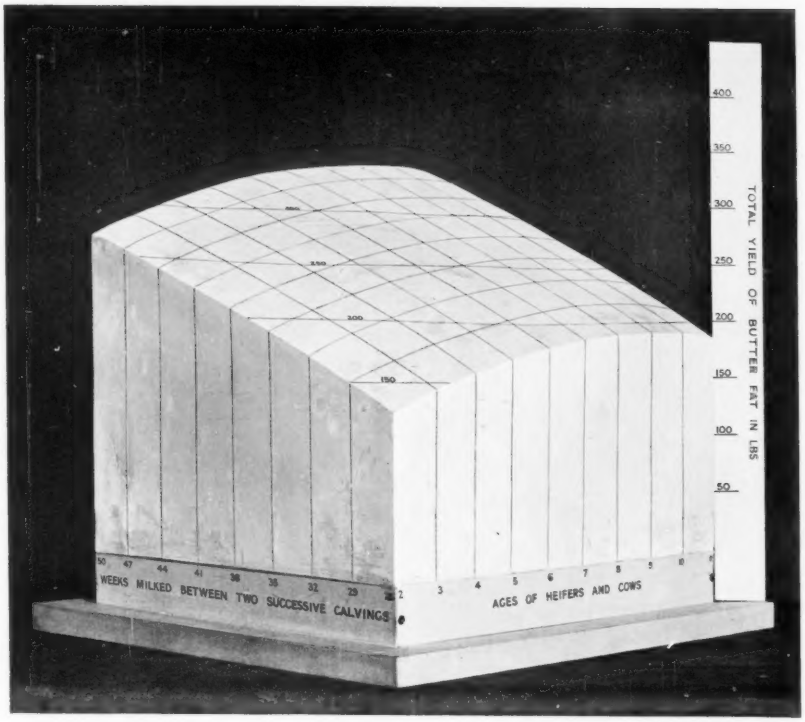
(5) *Study of the Nature of the Classification practised by the Association.*

(*a*) Present Classification.

We shall now consider, in the 1912 data, the nature of the classification made by the Association and the statistical differences between the classes created by that body and believed to be divisions of a practical nature. According to the Milk Records Association, the cows which fall into Class I and the heifers in Class II are regarded as "Good," cows in Class III and heifers in Class IV are regarded as "Intermediate," and cows in Class V and heifers in Class VI are considered "Bad." (See Section I (2), for the definitions of the six classes.)

In the records of 6648 cows and heifers, 4949 are classed as cows and 1699 are classed as heifers. The following table shows the numbers and percentages of good, intermediate and bad cows and heifers according to the Association's classification.

DIAGRAM XXVIII.



Photograph of a Plaster Model showing the Total Yield of Butter Fat for Cows of a given Age and given Length of Lactation Period.

TABLE XII.

Table showing the Average Yield of Butter Fat for a given Age of Cow and a given Length of Lactation Period.

	Weeks in Milk	Years of Age. 2	3	4	5	6	7	8	9	10	11
Roman type denotes means from two sets of parabolae	26	132·4 *131·5* **135·0**	147·3 *146·9* **157·9**	162·7 *160·0* **159·0**	171·5 *170·9* **166·4**	176·8 *179·5* **179·3**	184·0 *185·9* **186·4**	187·8 *190·0* **206·0**	184·7 *191·8* **179·1**	190·9 *191·4* **250·0**	192·5 *188·7* **216·7**
Italic type denotes means given by surface calculated from above means	29	155·8 *152·4* **162·4**	168·3 *168·1* **169·3**	183·7 *181·5* **186·6**	193·6 *192·2* **196·3**	200·0 *201·7* **198·8**	208·6 *208·3* **218·3**	212·7 *212·7* **202·9**	212·0 *214·9* **212·1**	219·1 *214·7* **229·0**	223·4 *212·4* **245·0**
Heavy type denotes observed values	32	178·5 *172·4* **189·4**	189·4 *188·5* **189·6**	204·8 *202·2* **204·0**	215·4 *213·7* **220·9**	221·8 *223·0* **221·1**	230·2 *229·9* **232·7**	232·8 *234·7* **227·9**	232·5 *237·1* **228·2**	237·9 *237·3* **245·8**	240·9 *235·2* **243·1**
	35	195·2 *191·7* **193·0**	205·9 *208·0* **204·0**	222·3 *222·1* **223·2**	234·6 *233·9* **232·8**	241·9 *243·5* **240·2**	251·0 *250·8* **248·0**	253·2 *255·8* **253·1**	255·0 *258·5* **253·8**	259·5 *259·1* **255·1**	261·9 *257·3* **258·8**
	38	213·0 *210·1* **208·3**	223·7 *226·8* **221·0**	240·8 *241·2* **240·1**	254·4 *253·3* **254·9**	262·1 *263·2* **261·6**	271·4 *270·8* **270·3**	272·6 *276·1* **266·6**	276·5 *279·2* **271·4**	279·5 *280·0* **275·9**	280·0 *278·5* **268·5**
	41	228·8 *227·7* **234·1**	240·0 *244·7* **234·7**	258·1 *259·4* **260·3**	273·4 *271·9* **272·5**	281·7 *282·0* **282·9**	291·8 *289·9* **290·0**	292·6 *295·6* **287·3**	299·9 *299·0* **298·0**	302·3 *300·1* **300·8**	301·5 *299·0* **293·6**
	44	246·1 *244·6* **243·6**	257·9 *261·8* **257·3**	276·5 *276·8* **276·5**	293·1 *289·6* **301·2**	301·8 *300·1* **308·7**	311·8 *308·3* **304·9**	311·8 *314·3* **311·3**	322·5 *318·0* **322·4**	323·9 *319·4* **324·1**	320·6 *318·6* **325·0**
	47	259·6 *260·6* **246·5**	273·7 *278·2* **274·8**	293·8 *293·5* **296·5**	312·1 *306·5* **321·3**	320·9 *317·3* **311·3**	330·4 *325·8* **338·9**	328·7 *332·2* **312·7**	342·1 *336·2* **370·0**	340·6 *337·9* **340·0**	332·1 *337·4* **330·0**
	50	270·6 *275·8* **260·9**	288·9 *293·7* **295·1**	311·3 *309·3* **306·0**	331·3 *322·7* **351·0**	339·0 *333·8* **330·0**	346·3 *342·6* **348·0**	340·3 *349·2* **324·6**	353·4 *353·5* **370·0**	344·7 *355·6* **270·0**	325·6 *355·4* **370·0**

TABLE XIII.

1912 *Data.*

	Cows		Heifers	
	Number	Percentage	Number	Percentage
Good	2490	50·31	1148	67·57
Intermediate	2252	45·51	496	29·19
Bad	207	4·18	55	3·24
Totals	4949	100·00	1699	100·00

It will be seen that 50 per cent. of the cows belong to the good class, and over 45 per cent. belong to the intermediate class, while about 4 per cent. are classed as bad. About 68 per cent. of the heifers are classed as good, about 29 per cent. as intermediate, and over 3 per cent. as bad. On examination of the table below (Table XIV), however, it is clear that at least 7 per cent. of the intermediate cows should have been classed as bad and a few should have been classed as good. A small number of good cows should have been placed in the intermediate class.

TABLE XIV.

Percentage of Cows and Heifers in each Class—1912 Data.

Yield of Butter Fat in pounds	Cows						Heifers					
	Good		Intermediate		Bad		Good		Intermediate		Bad	
	No.	Per cent.	No.	Per cent.	No.	Per cent.	No.	Per cent.	No.	Per cent.	No.	Per cent.
110	—	—	37	1·64	13	6·28	—	—	34	6·85	28	50·91
130	—	—	53	2·35	53	25·60	—	—	46	9·27	24	43·64
150	—	—	62	2·75	102	49·28	—	—	95	19·16	2	3·63
170	1	·04	280	12·43	39	18·84	—	—	140	28·23	—	—
190	—	—	437	19·41	—	—	13	1·13	173	34·88	1	1·82
210	5	·20	543	24·12	—	—	285	24·83	5	1·01	—	—
230	6	·24	591	26·24	—	—	241	20·99	1	·20	—	—
250	395	15·86	245	10·88	—	—	198	17·25	1	·20	—	—
270	616	24·74	3	·13	—	—	142	12·37	—	—	—	—
290	467	18·76	—	—	—	—	108	9·41	1	·20	—	—
310	340	13·66	1	·05	—	—	71	6·18	—	—	—	—
330	238	9·56	—	—	—	—	35	3·05	—	—	—	—
350	152	6·10	—	—	—	—	28	2·44	—	—	—	—
370	103	4·14	—	—	—	—	12	1·04	—	—	—	—
390	58	2·33	—	—	—	—	6	·52	—	—	—	—
410	35	1·41	—	—	—	—	5	·44	—	—	—	—
430	30	1·20	—	—	—	—	3	·26	—	—	—	—
450	17	·68	—	—	—	—	1	·09	—	—	—	—
470	27	1·08	—	—	—	—	—	—	—	—	—	—
	2490	100·0	2252	100·0	207	100·0	1148	100·0	496	100·0	55	100·0

The frequencies in this Table are shown graphically in Diagram XXIX for Cows and in Diagram XXX for Heifers.

A similar phenomenon is observed with respect to the classification of heifers. As many as 16 per cent. of intermediate heifers should be classed as bad and a few should be classed as good. The analysis, however, has been made in accordance with the figures of the Association. It will be seen that in each of the three classes there is great variability in the total yield of butter fat (number of gallons of milk containing 1 per cent. butter fat). Of the good cows, 59·36 per cent. give a total yield of butter fat for a lactation period of between 240 and 300 pounds, 35·78 per cent. between 300 and 400 pounds and 4·38 per cent. give about 400 pounds. Table XIV above shows the number and percentages of cows and heifers for class units of 20 pounds in the total butter fat distribution.

DIAGRAM XXIX.

Total Yield of Butter Fat. Good, Bad and Intermediate Cows—1912.

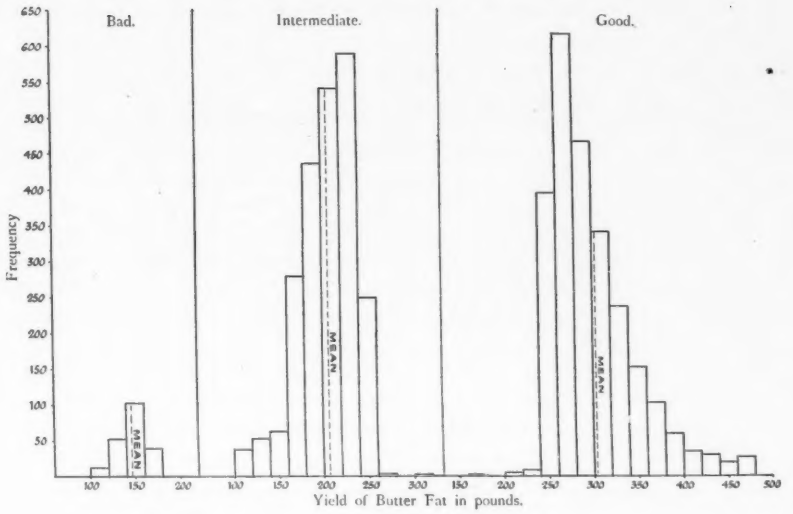
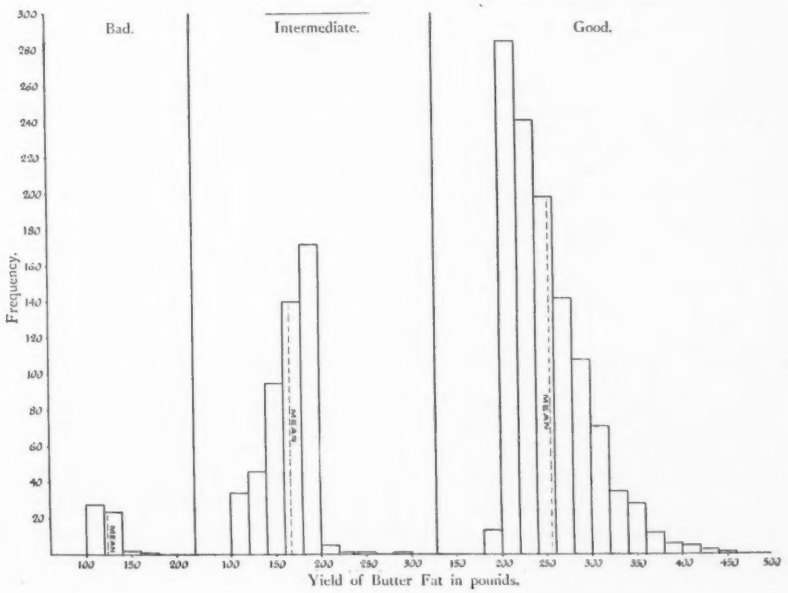


DIAGRAM XXX.

Total Yield of Butter Fat. Good, Bad and Intermediate Heifers—1912.



The means, standard deviations and coefficients of variation in each of the three classes for the four characters, yield of butter fat, length of lactation period, percentage of butter fat and quantity, may now be considered. Table XV below shows the results:

TABLE XV.

Means, Standard Deviations and Coefficients of Variation.

1912 *Data.*

Quantity.

	Good		Intermediate		Bad	
	Cows	Heifers	Cows	Heifers	Cows	Heifers
No. observed	2490	1148	2252	496	207	55
Mean	804·16	654·22	568·18	436·09	418·96	306·82
S.D.	136·53	116·86	94·28	79·30	58·97	49·88
C.V.	16·98	17·86	16·59	18·18	14·08	16·26

Fat Percentage.

	Good		Intermediate		Bad	
	Cows	Heifers	Cows	Heifers	Cows	Heifers
No. observed	2490	1148	2252	496	207	55
Mean	3·76	3·89	3·63	3·83	3·50	3·86
S.D.	0·32	0·34	0·33	0·35	0·35	0·35
C.V.	8·51	8·74	9·09	9·14	10·00	9·07

Length of Lactation Period.

	Good		Intermediate		Bad	
	Cows	Heifers	Cows	Heifers	Cows	Heifers
No. observed	2490	1148	2252	496	207	55
Mean	39·83	41·68	34·21	33·38	30·91	28·29
S.D.	6·16	6·53	4·83	5·33	4·64	5·60
C.V.	15·43	15·67	14·12	15·97	15·01	19·79

Total Yield of Butter Fat.

	Good		Intermediate		Bad	
	Cows	Heifers	Cows	Heifers	Cows	Heifers
No. observed	2490	1148	2252	486	207	55
Mean	300·63	253·34	205·69	166·25	146·14	121·64
S.D.	46·44	43·09	30·99	25·74	16·23	14·62
C.V.	15·45	17·01	15·07	15·48	11·11	12·02

It will be seen that the mean difference in yield of butter fat between good and intermediate cows amounts to 94·94 pounds, and the mean difference between bad and intermediate cows is 59·55 pounds. The corresponding figures for heifers are 87·09 pounds and 44·61 pounds respectively. The distributions of yield of butter fat for good, bad and intermediate cows and heifers respectively, together with the means, are shown in Diagrams XXIX and XXX. It will be observed that there is very little difference between the *mean percentages of butter fat* among the three classes of heifers. There is only a mean difference of 0·13 between good and intermediate cows and the same between intermediate and bad cows. Striking differences, however, occur in the mean length of lactation period and in the mean quantity of milk for the three classes. With a long mean lactation period (40 weeks) good cows have a mean yield of 804 gallons; intermediate cows, 34 weeks and 568 gallons; and bad cows, 31 weeks and 419 gallons. There is the least relative variability in the percentage of butter fat—the coefficient of variation runs from 8 to 10 for all classes. In yield of butter fat, bad cows and heifers are relatively less variable than intermediate and good cows and heifers. Bad heifers have the greatest relative variability in the length of the lactation period. Heifers of all classes have a slightly greater relative variability than cows of all classes in total yield of milk (quantity in gallons) during a lactation period.

(*b*) Classification of Cows.

(1) It is clear that the division of cows and heifers into good, intermediate and bad, without reference to age of cow, and duration of lactation period, to mention only two factors, is an arbitrary division. This arises from the fact that a cow might be classed as intermediate because of its age and probably shorter lactation period, when it would really be classed as good when it reaches its maximum production. Take a simple example, as seen in the following table (Table XVI) which gives the mean production of total butter fat for various age groups.

TABLE XVI.

Age group	Mean Production of Butter Fat during lactation period (pounds)
2—3	223·3
4—8	240·8
9—18	263·4

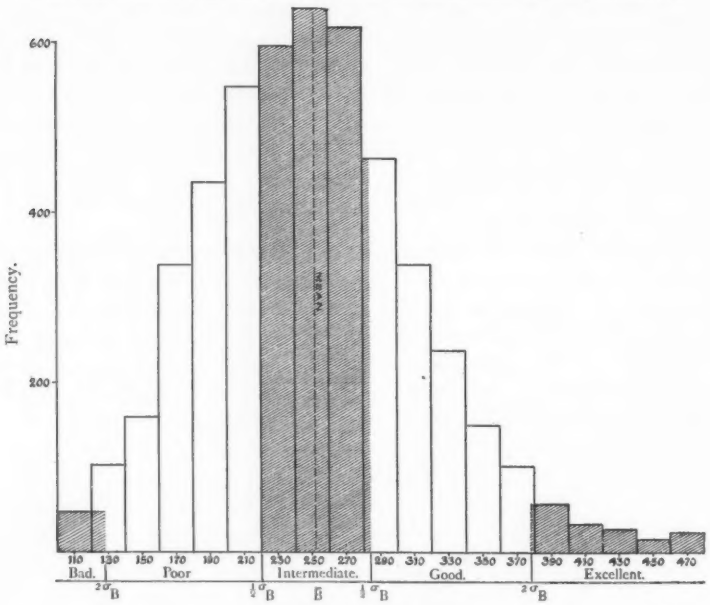
Cows ought to be selected for maximum milk production at certain ages and only those which have long lactation periods. What we want to know in order to make a scientific classification into good, intermediate and bad is the variation in total yield of butter fat for constant lactation period and a constant age of cow. Such variations are considered under Section II (4). It therefore appears to the writer that the classification of greatest practical value to the dairyman must be

based on a table showing mean yields of butter fat for various lengths of lactation period and for various ages of cow. (See Table XII.) The author recommends this method.

(2) If, however, the Association decides to continue measuring without reference to age or duration of lactation period, the milk yielding qualities of a cow by quantity only, in gallons of milk containing one per cent of butter fat or, what is the same thing as the writer expresses it, the total yield of butter fat during a lactation period*, it seems necessary, from the results, to reconsider the Association's method of division into good, intermediate and bad cows and heifers.

DIAGRAM XXXI.

Total Yield of Butter Fat. Cows only—1912 Data. Classification of Cows, Method (*a*).



Yield of Butter Fat in pounds.

The writer has considered three separate methods all based upon the variability of the distribution of total butter fat as measured by the standard deviation.

Method (*a*). This method is based upon the idea of regarding as intermediate cows those approximating the mean value, the range being from $(B-\frac{1}{2}\sigma_B)$ to $(B+\frac{1}{2}\sigma_B)$. These cows are truly intermediate in yield of butter fat. Those cows

* If B = number of pounds of butter fat during a lactation period, V = number of gallons of milk containing 1 per cent. butter fat, and $W = 10V$ = corresponding number of "pounds" of milk containing 1 per cent. of butter fat, then $B = \frac{1}{100} \times W$.

which are better than intermediate and which fall between $(B + \frac{1}{2}\sigma_B)$ and $(B + 2\sigma_B)$ can be regarded as good, while those above $(B + 2\sigma_B)$ are excellent cows. Those which fall below the intermediate class and lie between $(B - \frac{1}{2}\sigma_B)$ and $(B - 2\sigma_B)$ must be regarded as poor cows, while those beyond $(B - 2\sigma_B)$ are certainly bad cows. This method has been applied to 4949 cows and 1699 heifers separately with the following results (see Diagram XXXI and Table XVII).

If "good + excellent" cows are considered together as one group and the "poor + bad" cows are considered as another group, we have now three divisions. It is seen that about 29 per cent. of the cows are "good + excellent," 39 per cent.

TABLE XVII.

Class of Cow	Range of Yield of Milk containing 1 per cent. butter fat (gallons)	No. of Cows	Percentage
(1) Bad	Below 1220	60	1·2
(2) Poor	1220—2190	1536	31·0
(3) Intermediate	2190—2830	1928	39·0
(4) Good	2830—3800	1248	25·2
(5) Excellent	Above 3800	177	3·6
Total		4949	100·0

Class of Heifer	Range of Yield of Milk containing 1 per cent. butter fat (gallons)	No. of Heifers	Percentage
(1) Bad	Below 1070	36	2·1
(2) Poor	1070—1940	544	32·0
(3) Intermediate	1940—2530	604	35·6
(4) Good	2530—3400	436	25·6
(5) Excellent	Above 3400	79	4·7
Total		1699	100·0

are intermediate and 32 per cent. are "poor + bad." There are 30 per cent. "good + excellent" heifers, 36 per cent. "intermediate" heifers and 34 per cent. "poor + bad" heifers. From the point of view of the dairyman I think it is better to have a five-fold classification rather than a three-fold classification. This enables him to discriminate between poor and bad cows and between good and excellent cows. A similar method can be adopted for heifers.

Method (*b*). All cows falling between B and $(B + \sigma_B)$ are classed as intermediate as are also those lying between B and $(B - \sigma_B)$. Those lying between $(B + \sigma_B)$ and $(B + 2\sigma_B)$ are classed as good, while those beyond $(B + 2\sigma_B)$ are classed as excellent. All cows between $(B - \sigma_B)$ and $(B - 2\sigma_B)$ are classed as poor, and those beyond

($B - 2\sigma_B$) as bad. The distribution of total butter fat for heifers is treated in a similar manner. The following table (Table XVIII) shows the approximate numbers and percentages falling into these classes (see also Diagrams XXXII and XXXIII). It will be seen from the table that the percentages in each class are practically the same for both cows and heifers. This remarkable agreement shows the validity of taking the standard deviation as the unit in each case. Of course the standard deviation in the case of cows is 64, while in the case of heifers it is 58 pounds.

TABLE XVIII.

1912 *Data.*

Class of Cow	Range of Yield of Milk containing 1 per cent. butter fat (gallons)	No. of Cows	Percentage	
(1) Bad	Below 1220	66	1·3	
(2) Poor	1220—1860	727	14·7	
(3) Intermediate	1860—2510	1749	35·3	68·9
(4) Intermediate	2510—3150	1662	33·6	
(5) Good	3150—3800	573	11·6	
(6) Excellent	Above 3800	172	3·5	
Total		4949	100·0	

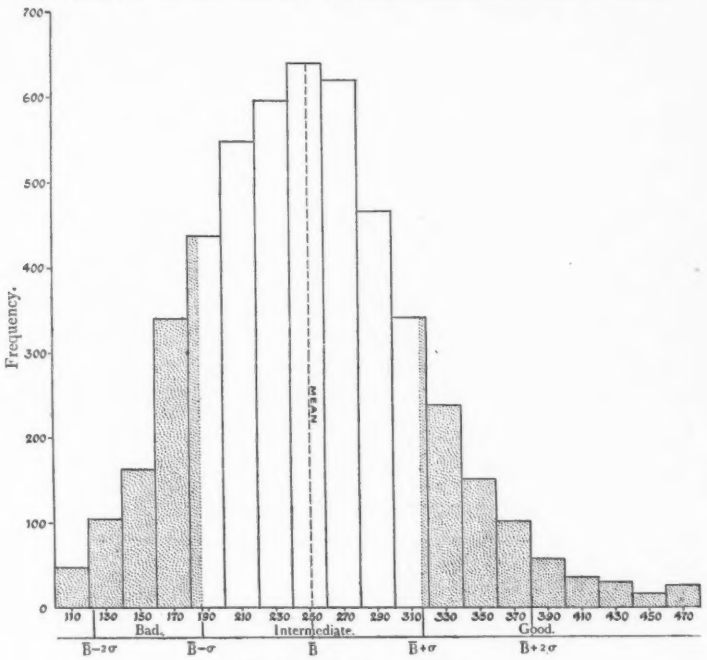
Class of Heifer	Range of Yield of Milk containing 1 per cent.. butter fat (gallons)	No. of Heifers	Percentage	
(1) Bad	Below 1070	25	1·5	
(2) Poor	1070—1650	246	14·5	
(3) Intermediate	1650—2230	623	36·7	68·8
(4) Intermediate	2230—2820	546	32·1	
(5) Good	2820—3400	204	12·0	
(6) Excellent	Above 3400	55	3·2	
Total		1699	100·0	

If it seemed desirable to unite classes 3 and 4 into one class, this would have the effect of increasing the intermediate class. Uniting classes 1 and 2 as poor, and classes 5 and 6 as good, will give a three-fold classification: see Table XVIII. The range would now be from ($B - \sigma_B$) to ($B + \sigma_B$) for the intermediate, and we should have 69 per cent. intermediate cows, 16 per cent. good cows; 15 per cent. bad cows. The adoption of the five-fold method would create a greater stringency in getting into the good and excellent classes. The poor class would also be reduced while the bad class would remain the same.

Method (*c*). We can consider the whole distribution of cows and heifers as one group. It should be pointed out that it is as rational to do this as to take cows of various ages and heifers of various ages and divide them separately into good, intermediate and bad. Such a division is shown in Diagram XXXIV. This method, however, would classify as bad with respect to butter fat certain poor cows because of the shorter lactation period and because of age, as well as other poor cows which are poor in themselves irrespective of duration of lactation period and age.

DIAGRAM XXXII.

Total Yield of Butter Fat. Cows only—1912 Data. Classification of Cows, Method (*b*).

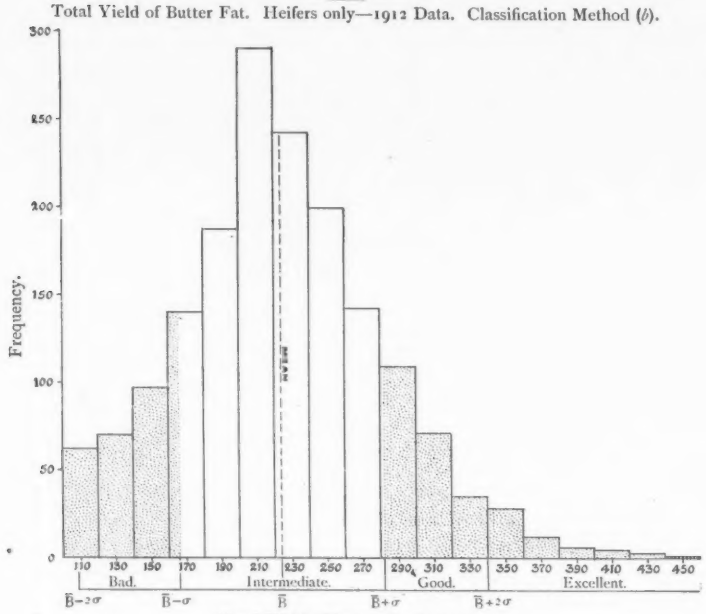


Yield of Butter Fat in pounds.

The effect of considering cows and heifers together is seen in Table XIX where the means, standard deviations and coefficients of variation are given. The difference in total yield of butter fat between good and intermediate classes is now seen to be 87 pounds. The difference in quantity between good and intermediate is 212·5 gallons and the corresponding difference between intermediate and bad is 149·11 gallons. It is to be observed that this classification has practically no effect on the mean percentage of butter fat.

DIAGRAM XXXIII.

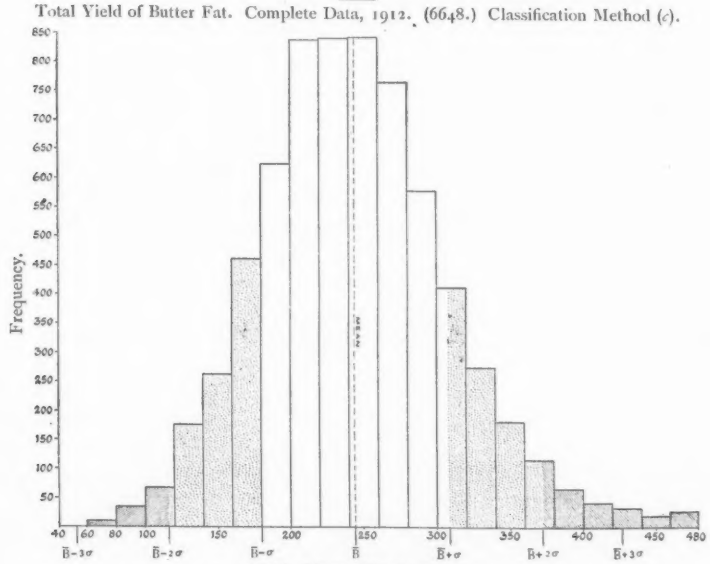
Total Yield of Butter Fat. Heifers only—1912 Data. Classification Method (*b*).



Total Yield of Butter Fat in pounds.

DIAGRAM XXXIV.

Total Yield of Butter Fat. Complete Data, 1912. (6648.) Classification Method (*c*).



Yield of Butter Fat in pounds.

TABLE XIX

Cows and Heifers together—1912 Data.

Quantity.

Class	Number Observed	Mean	Standard Deviation	Coefficient of Variation
All Cows and Heifers	6648	654·20	172·01	26·29
Good	3638	756·84	147·22	19·45
Intermediate	2748	544·34	104·88	19·27
Bad	262	395·23	73·69	18·65

Percentage.

Class	Number Observed	Mean	Standard Deviation	Coefficient of Variation
All Cows and Heifers	6648	3·74	0·35	9·28
Good	3638	3·80	0·33	8·73
Intermediate	2748	3·67	0·34	9·38
Bad	262	3·58	0·38	10·61

Length of Lactation Period.

Class	Number Observed	Mean	Standard Deviation	Coefficient of Variation
All Cows and Heifers	6648	37·41	6·67	17·83
Good	3638	40·49	6·33	15·64
Intermediate	2748	34·06	4·93	14·49
Bad	262	30·36	4·98	16·39

Yield of Butter Fat.

Class	Number Observed	Mean	Standard Deviation	Coefficient of Variation
All Cows and Heifers	6648	243·77	64·39	26·42
Good	3638	285·71	50·45	17·66
Intermediate	2748	198·57	33·71	16·98
Bad	262	140·99	18·77	13·32

(*c*) *Correlations of characters in the Good, Intermediate and Bad Classes.*

We shall now consider the variation produced in the correlations between the various characters by dividing the data according to the classification practised by the Association. We shall use η for comparative purposes as this is a more accurate

measure of the correlation. The following table (Table XX) gives the values of this constant and also the values of r. It will be seen that in most cases the value of η is less for the whole population than for the "good," "intermediate" and "bad" classes. In the case of quantity and lactation period the whole population shows a higher correlation than the three separate classes. The correlation r_{QW} for good cows is lower than the corresponding correlation for the other two classes, indicating that "good" cows are less dependent on the duration of the lactation period for yield than the "intermediate" and "bad" cows. Between age and lactation period

TABLE XX.

Correlation Coefficients and Correlation Ratios—1912 *Data.*

	All Cows	Good	Intermediate	Bad
r_{AQ}	+·3418	+·4249	+·4202	+·4504
$\eta_{Q.A}$	·3656	·4969	·5339	·6568
$\eta_{A.Q}$	·3487	·4753	·4371	·4669
r_{AP}	−·2336	−·2283	−·2749	−·3102
$\eta_{P.A}$	·2693	·2570	·3033	·4761
$\eta_{A.P}$	·2417	·2385	·2861	·3268
r_{AW}	−·0379	−·1321	+·0385	+·0882
$\eta_{W.A}$	·1360	·1431	·0978	·3115
$\eta_{A.W}$	·1315	·1686	·1137	·1975
r_{AB}	+·2650	+·3594	+·3383	+·3528
$\eta_{B.A}$	·2858	·4464	·4750	·5588
$\eta_{A.B}$	·2709	·4517	·3577	·3751
r_{PQ}	−·1625	−·4085	−·4556	−·6287
$\eta_{Q.P}$	·1809	·4150	·4583	·6444
$\eta_{P.Q}$	·1726	·4661	·4822	·6736
r_{PW}	+·0540	−·0273	−·1170	−·2065
$\eta_{W.P}$	·0763	·0779	·1448	·2769
$\eta_{P.W}$	·0990	·0794	·1400	·2656
r_{QW}	+·6139	+·3845	+·5248	+·4314
$\eta_{W.Q}$	·6300	·4172	·5617	·4470
$\eta_{Q.W}$	·6205	·3961	·5632	·5068

a small negative correlation is indicated in the whole population, while in the three classes the "good" cows show a slightly larger negative correlation, and the "intermediate" and "bad" cows give a small positive value for r. Among the classes as a whole there is a tendency for the various characters to be more highly correlated, whether negatively or positively, in the "bad" class than in the other two. In the majority of cases the intermediate class shows a correlation lying between that for the good and bad classes. It is seen that the correlation between P and Q is negative in character and that, for the group of bad cows, the negative correlation

is very high, indicating that any increase in quantity is accompanied by a higher drop in the percentage of fat in this group than in the other two groups. The value of r_{PQ} for the whole population is a low negative quantity, indicating that, in general, an increase in quantity is accompanied by a slight decrease in the percentage of butter fat during a lactation period.

(*d*) *Results from Combined Data for* 1911 *and* 1912.

A study of the 1911 and 1912 data was made in which the cows were divided into three age groups, namely, Group I (2—3 years inclusive); Group II (4—8 years inclusive); and Group III (9—18 years inclusive). Table XXI below shows the results of this analysis. Looked at from this standpoint it will be seen that there is a progressive increase with age in the mean quantity during a lactation period. Group II has a greater mean yield than Group I by 80 gallons, while Group III has a greater mean yield than Group II by 77 gallons. Group III, however, is the most variable group, and Group I is the least variable.

TABLE XXI.

Means and Standard Deviations—1911 *and* 1912 *Combined Data.*

Age Groups	*N*	Quantity (gallons)		Percentage of Fat		Weeks		Total Yield of Butter Fat (pounds)	
		Mean	S.D.	Mean	S. D.	Mean	S.D.	Mean	S. D.
I. 2—3 years inclusive*	2784	570·70	148·34	3·87	0·34	38·45	7·25	219·94	56·90
II. 4—8 years inclusive	6709	651·05	161·31	3·71	0·33	36·49	6·13	240·79	61·07
III. 9—18 years inclusive	1854	728·40	172·66	3·62	0·31	36·85	6·56	263·40	65·08
IV. 4—18 years inclusive†	8563	667·80	166·28	3·69	0·32	36·56	6·17	245·69	62·39

* Total Heifers. † Total Cows.

Of course selection again appears here as a factor, the good older cows (good from the point of larger yield) are kept for dairy purposes. The mean percentage of butter fat is highest in Group I and lowest in Group III, while Group I has the longest lactation period. The most instructive result is that which shows the mean total yield of butter fat for a lactation period in these three groups. The yield is lowest in Group I (220 pounds) and highest in Group III (263 pounds), a difference of 43 pounds in favour of the group of oldest cows.

When all cows are considered (4—18 years) it is seen from the table that the mean quantity in gallons for the whole group of cows is only slightly greater than the corresponding mean for Group II (4—8 years). The mean percentage of butter fat and the mean length of lactation period are practically the same as for the 4—8 years group. The mean yield of butter fat for all cows is only 5 pounds greater than the mean yield for cows in Group II.

A study of the correlations for various pairs of characters in each of the three age groups shows that, so far as age is concerned, the only significant correlations

are those between age and total yield of butter fat and age and quantity in Group II (4—8 years). The correlation between quantity and total yield of butter fat is high and is practically the same for the three groups (0·93). The correlation between length of lactation period and total yield of butter fat is highest in Group I (0·73) and is lowest in Group III (0·64), owing to the slight falling off in the mean percentage of butter fat among the older cows, and to the falling off in the mean length of lactation period among older cows. In Group I, where the cows are practically all the same age, there is a significant negative correlation between quantity and percentage of butter fat ($-0·165$). There is practically no correlation between these two factors in the other two groups. When the correlation between quantity and total yield of butter fat is considered for a constant length of lactation period, it is found that practically the same value is obtained in the three groups, namely, 0·85, 0·89, and 0·88. The table below (Table XXII) shows the values of the correlation coefficients for the three groups.

TABLE XXII.

Correlation Coefficients—1911 *and* 1912 *Combined Data.*

Characters	Group I (2—3 years)	Group II (4—8 years)	Group III (9—18 years)
Age and Quantity	—	·2606	−·0415
Age and Butter Fat	—	·2110	−·0371
Age and Weeks	—	·0411	−·0267
Age and Percentage of Fat	—	−·1227	·0084
Quantity and Percentage	−·1653	·0932	−·0077
Quantity and Weeks	·7232	·6431	·6735
Quantity and Butter Fat	·9320	·9356	·9213
Weeks and Butter Fat	·7322	·6467	·6373
Butter Fat and Quantity (for a constant number of weeks)	·8533	·8898	·8779

The combined data 1911—1912 for heifers (Group I) were now studied with respect to yield. The heifers were divided with regard to yield into good, intermediate, and bad classes in accordance with the class limits of the Milk Records Association. The following table (Table XXIII) shows the results of this analysis

TABLE XXIII.

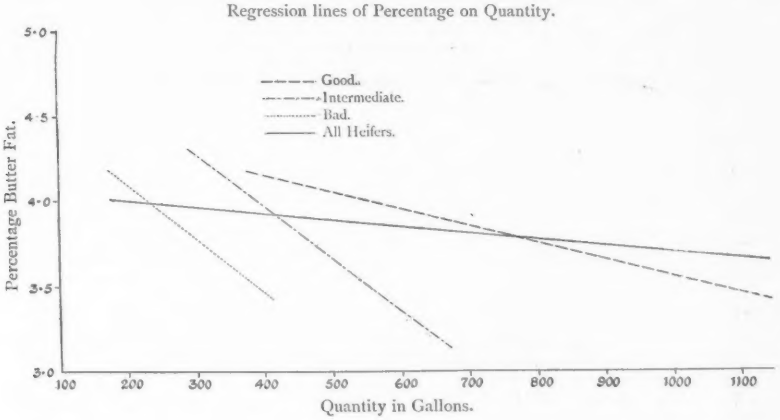
Heifers—1911 *and* 1912 *Combined Data.*

	N	Quantity (gallons)		Percentage of Fat		Weeks		Total Yield of Butter Fat (pounds)	
		Mean	S. D.	Mean	S. D.	Mean	S. D.	Mean	S. D.
Good	1802	648·97	113·86	3·89	0·33	41·36	6·43	251·65	41·53
Intermediate	822	452·46	59·42	3·82	0·34	34·34	4·51	171·36	18·40
Bad	160	298·75	49·78	3·82	0·36	26·84	5·34	113·38	16·88

for the four characters studied. There are striking differences in the mean quantities of the three classes but there is very little difference in the mean percentage of butter fat. There are striking differences in the mean length of lactation period and in the mean total yield of butter fat. The mean difference between the good and intermediate classes for yield of butter fat is found to be 80 pounds compared with 87 pounds for the 1912 data, while the mean difference between intermediate and bad for the same character is 58 pounds compared with 45 pounds for the 1912 data. When we consider quantity, the mean difference between good and intermediate is 197 gallons for the 1911 data compared with 218 gallons for the 1912 data, and the mean difference between intermediate and bad is 145 gallons compared with 129 gallons for 1912. The mean difference in length of lactation period between good and intermediate is 7 weeks compared with 8 weeks for the 1912 data, while the corresponding differences between intermediate and bad are 7·5 weeks and 5 weeks respectively.

DIAGRAM XXXV.

Regression lines of Percentage on Quantity.



The effect on the relationship between quantity and percentage, of dividing heifers into good, intermediate, and bad classes, can be seen by a study of Diagram XXXV. The correlation between quantity and percentage of butter fat is seen to be least in the case of the good class and greatest in the case of the intermediate class. It should first be noted that for all heifers r is $-$ ·1653, indicating that with an increase in quantity the percentage of butter fat on an average tends to decrease (see also pages 152 and 153). When the heifers are separated into the three quantity classes of the Association, it is found that the average percentage of butter fat associated with quantity rapidly decreases with increased quantity and the same holds good with regard to the intermediate class. When the relationship between percentage and number of weeks in milk is considered (Diagram XXXVI) it is seen that for the bad and intermediate classes there is a distinct tendency for the

DIAGRAM XXXVI.

Regression lines of Percentage on Weeks.

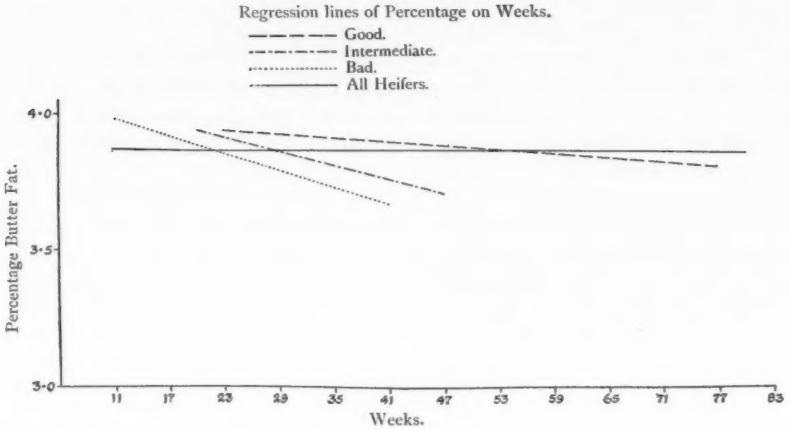
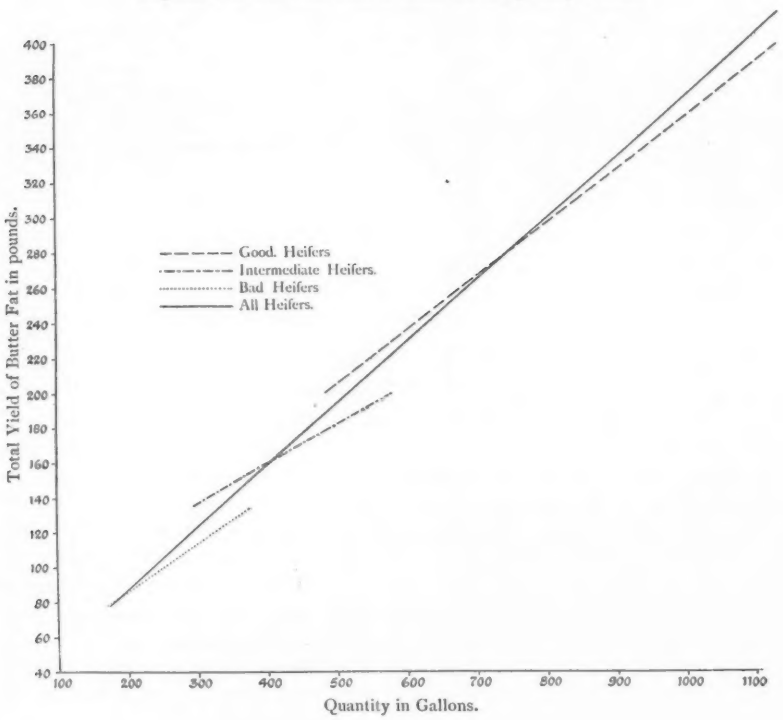


DIAGRAM XXXVII.

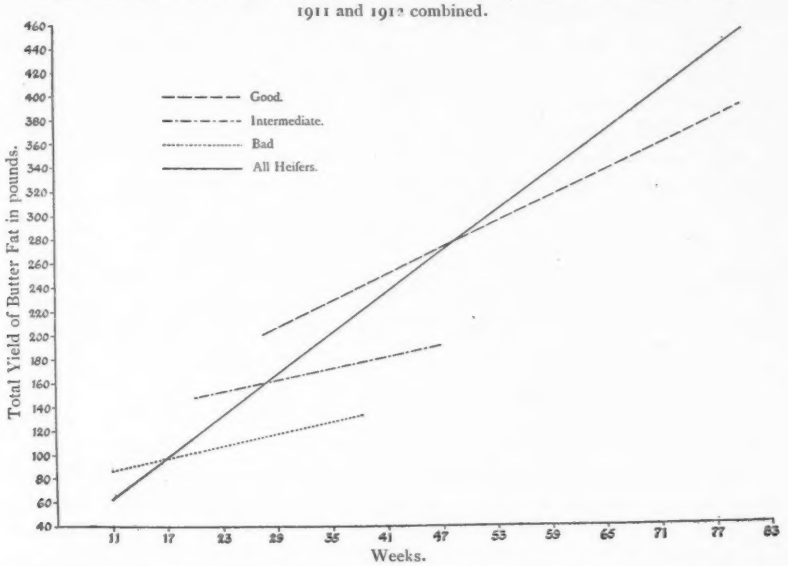
Regression lines of Total Yield of Butter Fat on Quantity of Milk.



average percentage of butter fat to fall the longer the lactation period. When all heifers are considered together, there is no appreciable difference in the average percentage of butter fat for varying lengths of lactation period. In other words, the length of lactation period does not affect the average percentage of butter fat given by any heifer. It is seen that bad and intermediate heifers have shorter lactation periods and that the average percentage of butter fat does depend to a slight degree on the length of the period. When total yield of butter fat is considered with respect to yield of milk, r is found to be 0·93 for all heifers while the corresponding correlations for the three classes are slightly less, that of the intermediate class being lowest, namely, 0·74 per cent. A reference to Diagram XXXVII shows the

DIAGRAM XXXVIII.

Regression lines of Total Yield of Butter Fat on Weeks. Good, Bad and Intermediate Heifers. 1911 and 1912 combined.



nature of the trend for the three classes and the general trend for all heifers. When the total yield of butter fat is considered with respect to length of lactation period it is found that for all heifers the correlation is highest, namely, 0·73. The correlations are considerably reduced when the heifers are divided into good, intermediate, and bad classes. The intermediate class has the lowest correlation, namely, 0·41; that is to say, the duration of the lactation period has least effect on total yield of butter fat in the intermediate class. For the good class the correlation is 0·58, showing that the good class is less affected by the length of lactation period than the intermediate class. In other words, a considerable number are giving high yield of butter fat for a shorter lactation period. The regression lines of total yield of butter fat on weeks are shown on Diagram XXXVIII.

(6) *Study of the Nature of Selection among Ayrshire Cows in the Records.*

(*a*) *Deduction of the Relationship between Quantity and Age in an unselected Population.*

In considering the problem of selection, it is necessary not only to study the nature of the distribution of quantity for 1911 and 1912, but also to consider the effect of a selected distribution of quantity upon the values of the correlations between age and quantity. It was found on examining the distribution of age both for 1911 and 1912 that a distinct drop in the numbers occurred in passing from three to four years of age. The explanation of this drop, as furnished by those engaged in the practice of dairying, is, that those heifers and young cows found to be poor milkers are sold and are therefore not kept for further milk recording. We have to bear in mind that we are dealing with recorded yields and there may be an indisposition on the part of dairy farmers to furnish records of low yielding cows. If that is the case, there must therefore be an absence of a number of low yielding cows in the records of the Association. Thus, the records of the Milk Records Association may not be a random sample of the whole of the cows in the country. They are selected in the sense that a certain number of low yielding cows may be excluded. Confine our attention, for instance, to a low yield of 250 to 300 gallons for a lactation period and we have the following numbers at the stated ages for 1911.

TABLE XXIV.

Number of Gallons	Years of Age				Total
	2	3	4	5	
250—300	5	10	2	2	19

Thus there is a total of 19 cows giving total yields corresponding to the mean value of 275 gallons for a lactation period. Is it reasonable to suppose that there will be no cows of six, seven, or eight years of age yielding up to 300 gallons? Is it reasonable to suppose that cows yielding from 250 to 300 gallons when young, will, when they reach five or six years of age, yield a sufficiently greater quantity of milk during a lactation period to place them in a category higher than 300 gallons? Of course the explanation is that these low yielding cows are sold off and do not appear in further records. Is it possible by modern statistical methods *to arrive at the true correlation between age and quantity* in an *unselected population*, from a knowledge of the correlation between these factors in a specially selected population? It is necessary to consider this problem because it might be contended that the whole of the relationship between age and quantity is due to the fact that mainly older cows are selected because of their higher yields and the poorer cows have been rejected. We must therefore endeavour to ascertain whether it can be established that there is a relationship between age and quantity, irrespective of

any action of selection. It may not be possible to give an exact figure to the degree of correlation between the two factors, but it will be sufficient for our purpose if we establish that there is naturally an increase in the average yield of milk as the cow gets older in an unselected population of cows.

The problem may be discussed in the following manner. Let r_{QA} = correlation coefficient for the whole unselected population; σ_Q = corresponding standard deviation of Q (yield); σ_A = corresponding standard deviation of A (age); $\overline{Q}$ = mean of Q distribution; $\overline{A}$ = mean of A distribution. Further, let ρ_{QA} = correlation coefficient from a table of values of Q and A, for values of Q upwards from a point below which selection as to yield is taking place and for all values of A for these values of Q; s_Q and s_A the corresponding standard deviations of Q and A, and $\tilde{Q}$ and $\tilde{A}$ the corresponding means. We can find ρ_{QA}, s_Q and s_A; the problem is to find r_{QA}, σ_Q and σ_A for the whole unselected population, and also to find $\overline{Q}$ and $\overline{A}$. The values of σ_Q and $\overline{Q}$ may be found by the method described by Pearson and Lee (*Biometrika*, Vol. VI. p. 59). Thus the unknown constants left are r_{QA}, σ_A and $\overline{A}$. The solution to the problem is found by applying the method described by Pearson (*Biometrika*, Vol. VIII. pp. 437—443) to this particular case. Let $\overline{A}'_Q$ = mean of

DIAGRAM XXXIX.

1911 Data.

Regression of Age on Quantity (from $Q = 625$ gallons upwards).

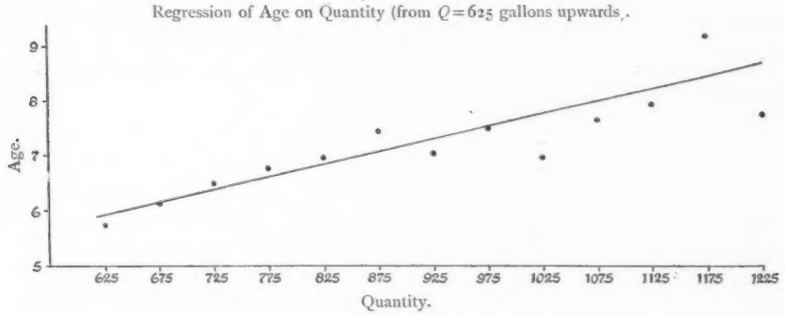
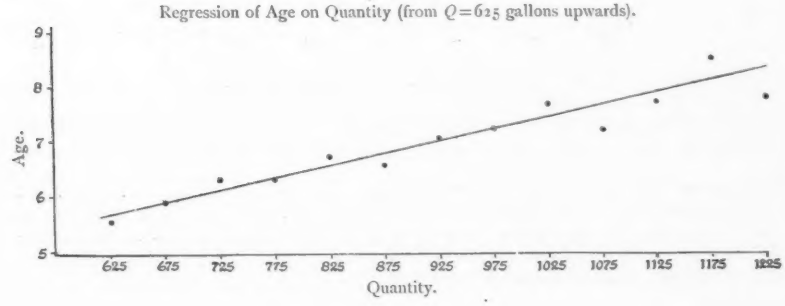


DIAGRAM XL.

1912 Data.

Regression of Age on Quantity (from $Q = 625$ gallons upwards).



A array for class Q in the curtailed distribution and A'_Q = any value of A in that array. The regression of A on Q is linear. (On applying the χ^2 test of Goodness of Fit, the value of P was found to be $P = \cdot 30$ for 1912—see Diagrams XXXIX and XL) and therefore, for the whole unselected population,

$$\bar{A}'_Q = \bar{A} + r_{QA}\frac{\sigma_A}{\sigma_Q}(Q - \bar{Q}).$$

We have therefore the following three equations necessary to find $\bar{A}$, σ_A and r_{QA}:

$$(1)\quad \bar{A} = \tilde{A} - r_{AQ}\frac{\sigma_A}{\sigma_Q}(\tilde{Q} - \bar{Q}),$$

$$(2)\quad r^2_{QA}\cdot\sigma^2_A = (s^2_A - \bar{s}^2_{AQ})\frac{\sigma^2_Q}{s^2_Q},$$

$$(3)\quad r^2_{QA}\cdot\sigma^2_A = \frac{s^2_A}{s^2_Q}\sigma^2_Q\rho^2_{QA}{}^*.$$

From (2) and (3) we get

$$\bar{s}^2_{AQ} = s^2_A(1 - \rho^2_{QA}).$$

Now for the whole unselected population the corresponding equation is known to be

$$\bar{\sigma}^2_{AQ} = \sigma^2_A(1 - r^2_{QA}).$$

Thus σ^2_A can be found and finally

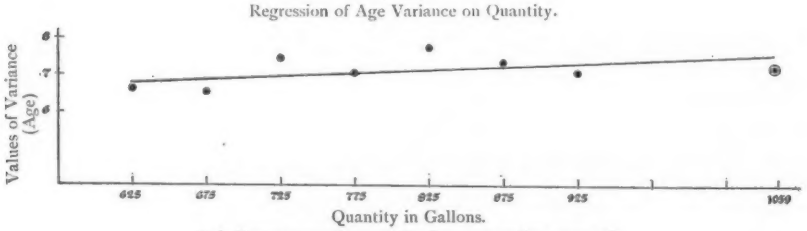
$$r^2_{QA} = 1\Big/\left\{1 + \frac{s^2_Q}{\sigma^2_Q}\cdot\frac{\bar{\sigma}^2_{AQ}}{(s^2_A - \bar{s}^2_{AQ})}\right\}.$$

It is assumed from Diagram XLI that the scedastic curve, σ^2_{AQ}, is linear with respect to Q, that is, $\sigma^2_{AQ} = a_0 + a_1Q$, and therefore

$$\bar{\sigma}^2_{AQ} = \frac{\Sigma(n_Q\sigma^2_{AQ})}{\Sigma(n_Q)} = a_0 + a_1\bar{Q}.$$

DIAGRAM XLI.

Regression of Age Variance on Quantity.



⊙ indicates mean for quantity from 950 gallons upwards.

As our object is not so much to determine the exact correlation between Q and A in an unselected population as to determine whether there is or is not in fact

* *Phil. Trans. Roy. Soc.* Series A, Vol. 200 (1902), pp. 1—66, "On the influence of natural selection on the variability and correlation of organs" (Pearson). See pp. 19 and 21, where we have the following theorem: If an organ has been modified only by indirect selection, then its partial regression coefficients on any complex of other organs, however large or small, provided it includes all the directly selected organs, will remain unchanged by the selection.

a significant correlation between an unselected Q and an unselected A, the Q distributed was truncated at $Q = 600$ gallons, below which there is certainly selection, and the various constants found. The values of the constants are given in Table XXV below. They show that Q is a function of age in an unselected population.

TABLE XXV

Values of Constants to find r_{QA}.

Constant	$\bar{Q}$	s_Q	$\bar{A}$	s_A	ρ_{QA}
Value Found	759·5	126·8	6·30	2·72	·2071
Constant	$\overline{Q}$	σ_Q	$\bar{\sigma}_{A_Q}{}^2$	σ_A	r_{QA}
Value Found	474·0	248·5	6·58	2·79	·3951

It is of course clear that, owing to the skewness of the distributions of Q and A in the selected population, any relationship between Q and A in the unselected population will be more or less non-linear in character. Hence the value of r_{QA} just deduced for the unselected population, being significantly greater than zero, is only of the nature of an average value.

(*b*) *Average Yield per single Milking contrasted with Average Yield per Milking for entire Lactation Period.*

It is, however, of interest to note that the data collected at the instance of the Scottish Interdepartmental Committee on Milk and analysed by the writer* are data from a practically unselected population, the cows being drawn at random from all parts of Scotland. But it should be remembered that (1) the quantities recorded were yields per single milking, (2) many of the cows were at the earlier stages of their lactation periods, and (3) the cows had calved at various months throughout the year. For such cows, unselected as they were for butter fat percentage, yield, breed and age, there is a distinct correlation between Q and A†. The regression is non-linear and the *particular* values of yield for various ages hold only for such a group of cows. *The fact however to be specially noted is the positive correlation which clearly proves that in an unselected population, Q is a function of age, a result which we have just deduced as highly probable from the selected population of the Scottish Milk Records Association's cows.*

As has just been stated, the yield in the case of the Interdepartmental data was yield per milking and not the average yield per week or per milking for an *entire lactation period*, which latter was the yield found from the Scottish Milk Records Association data. The *exact difference due to selection* between an unselected and a selected population cannot therefore be arrived at from the above two sets of data, although we know from the foregoing results that quantity of milk is a function of

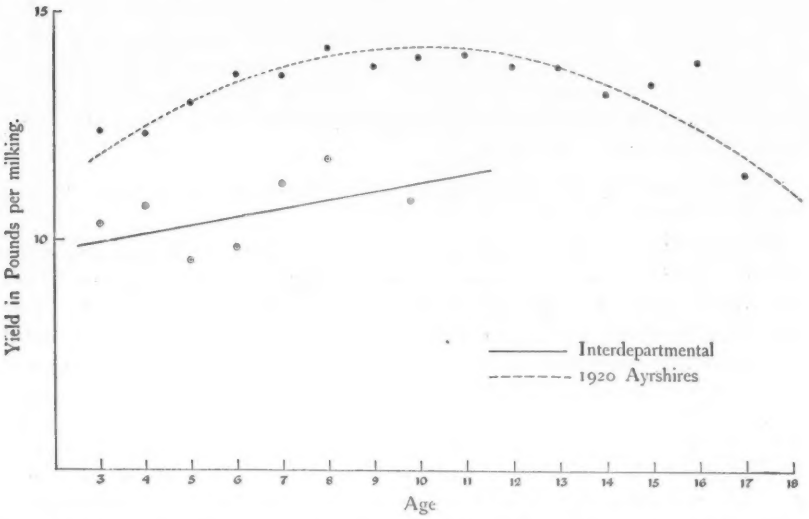
* "Variations in the Composition of Milk" (H.M. Stationery Office), p. 54.

† There is a falling off in yield among unselected cows after 7 years of age. Among selected cows, yield steadily increases up to 10 or 11 years of age.

age in both populations. And the reasons are perfectly clear. In the first place, daily yield per cow is also a function of the stage of lactation. It is seen, in the case of cows calving in April, to be highest shortly after calving and it falls gradually during the lactation period (see Diagrams LXXXII and LXXXIII). In the second place a reference to Table CXII, p. 174, Interdepartmental Monograph, shows that there is, in this group of cows, a relatively high number which have been a few weeks in milk. They therefore have relatively high daily yields. The corresponding proportion of cows a few weeks in milk is very much smaller in the Scottish Milk Records data. The proportions in the latter are very much higher at the later stages of the lactation period than the proportions present in the Interdepartmental data

DIAGRAM XLI (A).

Yields—Scottish Random Data and Milk Records Data contrasted.



(see Table LXVI of Appendix). And there are other reasons. Thus until we have data from both sources, of cows of the same breed, age, number of weeks in milk, and with approximately the same date of calving, no proper comparison can be instituted to show the *intensity of selection*. All we can do from our two sets of data is to show *by way of contrast* the average values of yield per milking for each age alongside the curve of average yield per milking over a whole lactation period from the Scottish Milk Records data. In the above diagram (Diagram XLI A) the average yield over an entire lactation period per milking for cows at each age from 3 to 18 years (S.M.R. data) is given and contrasted with the yield per single milking (Interdepartmental data). See also Table LXVIII of Appendix. Both for the Scottish Milk Records and the Interdepartmental data the cows had been in milk from 25 to 42 weeks. This range of weeks was chosen since, as will be seen from the

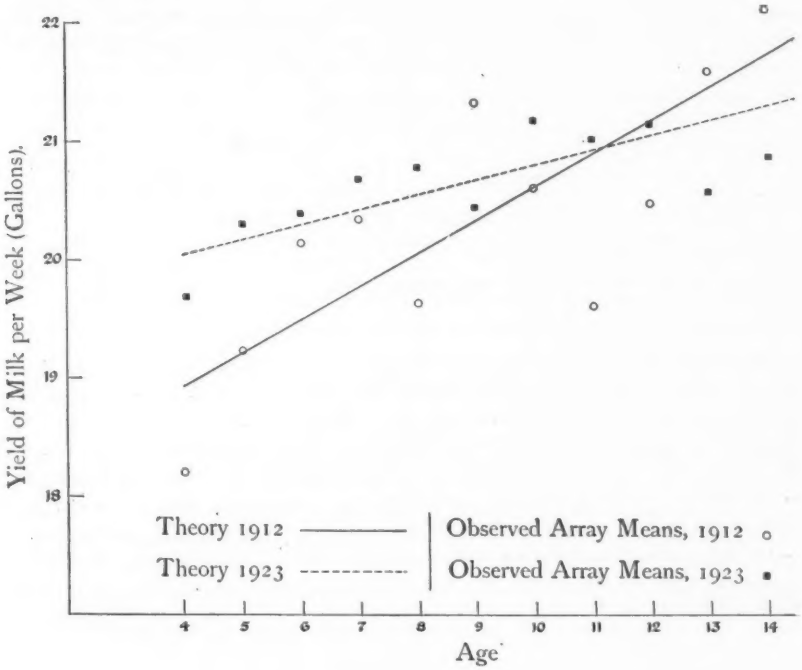
accompanying table (Table XXVI), cows with a complete lactation period of between 25 and 42 weeks and calving in various months of the year give a practically constant average yield per week. The diagram clearly shows that average yield per

TABLE XXVI.

Weeks in Milk	Average Yield per week (gallons)
26	18·5
29	18·4
32	18·7
35	18·5
38	18·8
41	18·8

DIAGRAM XLI (B).

Yield of Milk per Week for Cows of Various Ages in 1912 and 1923—Good Cows, i.e., Cows yielding above 250 pounds Butter Fat for Lactation Period.



single milking for cows of various ages, milked at a definite stage between 25 and 42 weeks from calving, cannot be accepted as a measure of the average yield per milking over the whole period of 25 to 42 weeks.

(*c*) *Selection for Yield between* 1911 *and* 1920.

While, as already stated, we cannot measure accurately from the data the difference between milk yield and the yield of butter fat in an unselected and the corresponding yields in a selected population, we can at any rate give a measure of the effect of selection between 1911 and 1920 for cows of all ages in the Milk Records population. The 1920 data for yields, however, as given in Table XXVIII are not directly comparable with those for 1911 since the recorders used the Association's "old method" of estimating yield in 1911, while in 1920 they used the "new method." It will be shown later (see p. 203) that by the recorders' use of the "old method" there is an average overestimate for the lactation period of 1·23 gallons or 0·162 per cent. of the average yield, whereas by the recorders' use of the "new method" there is an average overestimate of 26·07 gallons or 3·424 per cent. of the average yield (see page 202). The comparable 1920 yields were therefore determined by reducing the overestimate of 3·424 per cent. to an overestimate of 0·162 per cent. the latter figure being the overestimate by the old method as found from actual daily yields. The average lactation period in 1920 was 37·45 weeks, the corresponding figure for 1911 being 36·78 weeks. The corrected average yield for 1920 was therefore found for 36·78 weeks, for comparison with the 1911 yield. The following table gives the comparable figures for the two years 1911 and 1920.

TABLE XXVII.

Character	1911 (Total 4591)	1920 (Total 4912)
Average Yield of Milk for Lactation Period in gallons	634·8	662·9
Average Yield of Butter Fat for Lactation Period in pounds	236·1	252·5

This table shows that there is an increase of 28·1 gallons or 4·4 per cent. in the average yield of milk between 1911 and 1920. The increase in the yield of butter fat amounts to 16·4 pounds or 6·9 per cent. There is thus clear evidence from the data, of a progressive rise during the period from 1911 to 1920 in yield of milk and in total yield of butter fat among the herds of the Scottish Milk Records Association. It is, however, evident from these results that a complete statistical analysis of comparable data for *each year* from 1908 onwards to 1928 should be carried out in order that a quantitative measure of the extent of the improvement, due to selection at various ages, may be obtained. The results of such a statistical analysis would form a valuable guide to milk producers.

(*d*) *Selection of Good Cows for Yield between* 1912 *and* 1923.

The printed records of the Scottish Milk Records Association from 1924 onwards show only the records of good cows with a milk yield equivalent to not less than 2800 gallons containing one per cent. of milk fat. The year 1923 was the last year in which a list of good cows with a minimum yield of milk for a lactation period equivalent to 2500 gallons containing one per cent. of milk fat was printed in the Report on Milk Records. It is possible to measure the intensity of selection for yield among those good cows by comparing the average yield per week for all good cows for 1912 and 1923. It was found that 762 good cows appeared in the Report on Milk Records for 1912 as having completed lactation periods. The data were classified according to age of cow and yield per week both for 1912 and 1923. Two samples of 762 with the same age distribution as 1912 were taken at random from the 1923 records.

It was found that the mean yield per week for the 1912 group was 19·80. The corresponding mean yield for 1923, corrected to give comparable yields by the "old method" was 20·43, an improvement on the average of 0·63 gallon which is over three times the standard error between the two means. This difference must be regarded as significant. In other words, there has been a significant increase, on the average, in the yield of good cows from 1912 to 1923. Diagram XLI B (p. 163) shows the observed and theoretical results for both groups. It will be seen from the diagram and also from Table LXVII of Appendix that there has been an improvement by selection for yield for cows from 4 to 11 years of age. Here again it is desirable to emphasise the necessity of having the whole of the Scottish Milk Records tabled for each year in order to perform a statistical analysis for the purpose of revealing the extent of the improvement in yield and the increase, if any, in the percentage of butter fat.

III. Study of the 1920 Records and Comparisons with 1909 Records and other Data.

(1) *Means and Standard Deviations.*

The means and standard deviations of the various characters were determined and are given in the following table (Table XXVIII), together with the corresponding results for the years 1912, 1911 and 1908—1909. The results for the years 1908—1909 have been calculated from figures for the two years combined given by Pearl in the *Journal of Agricultural Research*, Vol. XVII. No. 6, already referred to in the introductory section.

It will be seen from the table that the average length of lactation period varies from 36·78 weeks to 37·45 weeks. The average percentage of butter fat during a lactation period rose from 3·69 in 1908—1909 to 3·81 in 1920. The figures for 1911 and 1912 were intermediate *between the two extremes.* Of far greater importance are the increases in the total yield of milk for a lactation period and in the total

TABLE XXVIII.

Character	1920 (Total 4912)		1912 (Total 6648)		1911 (Total 4591)		1908–9 (Total 6935)	
	Mean	S. D.	Mean	S. D.	Mean	S. D.	Mean	S. D.
1. Age in Years	6·22*	2·21	6·19	2·66	6·31	2·74	5·40	2·66
2. Length of Lactation Period in weeks	37·45	6·37	37·41	6·67	36·78	6·27	—	—
3. Percentage of Butter Fat	3·81	0·27	3·74	0·35	3·73	0·33	3·69	0·34
4. Yield of Butter Fat per week in pounds	7·09	1·32	6·52	—	6·42	—	—	—
5. Yield of Milk per week in gallons	18·61	3·37	17·49	—	17·26	—	16·49	3·33
6. Yield of Milk for Lactation Period in gallons	696·94	—	654·20	172·01	634·82	158·73	—	—
7. Yield of Butter Fat for Lactation Period in pounds	265·52	—	243·77	64·39	236·13	59·57	—	—

* No heifers were included in the 1920 data.

yield of butter fat for a lactation period. These are specially considered in the previous section (p. 164). The mean age for 1920 is 6·2 years, while the corresponding values for 1911 and 1912 are 6·31 and 6·19 respectively. In other words, the mean age of cow is practically constant for the period under review.

(2) *Frequency Distributions.*

(*a*) *General.*

Of the five characters studied, four were found to have distributions which were adequately represented by Type IV, Pearsonian curves. These four were: average percentage of butter fat during a lactation period, length of lactation

TABLE XXIX.

Values of Frequency Constants and Nature of Curves fitted to Distributions. (1920 *Data.*)

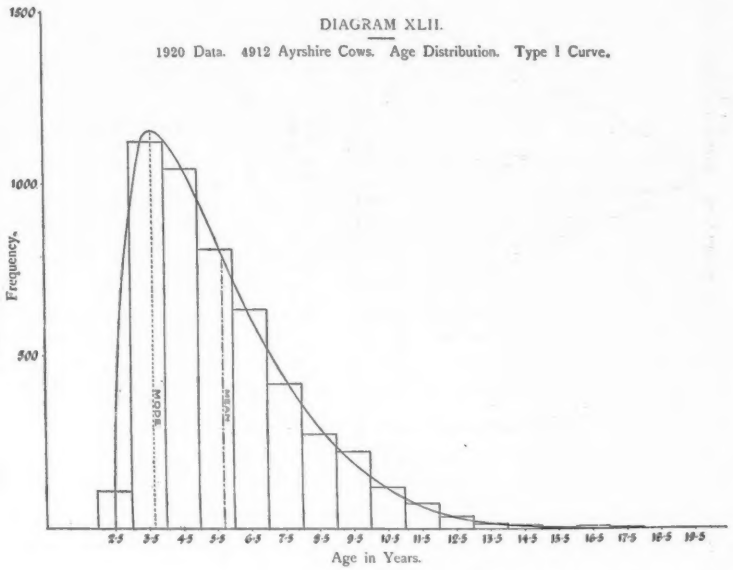
Character	N	Class Unit	μ_2	μ_3	μ_4	β_1	β_2	κ_2	Mean	Mode	Skewness	Curve fitted	P
Age in Years	4912	1	4·877	13·318	101·503	1·308	4·268	−·95	6·22	4·18	·93	Type I	·024
Average Percentage Butter Fat	4912	·25	1·201	·317	5·575	·058	3·867	·03	3·81	3·79	·08	Type IV	·019
Average Yield per Week in gallons	4912	1	11·375	12·705	428·708	·110	3·313	·28	18·61	18·10	·15	Type IV	·684
Length of Lactation Period in Weeks	4912	3	4·510	−4·279	78·390	·200	3·854	·14	37·45	38·52	−·17	Type IV	·815
Average Yield of Butter Fat per Week in pounds	4912	·5	6·952	7·316	166·857	·159	3·380	·44	7·09	6·85	·18	Type IV	·703

period, *average yield of milk per week during a lactation period for various ages,* and average yield of butter fat per week during a lactation period.

The distribution of age was found to belong to the group of curves known as Type I. Table XXIX shows the values of the frequency constants for age, weeks in milk, butter fat percentage, and average yield of butter fat per week, irrespective of age of cow (Diagrams XLII to XLVI).

DIAGRAM XLII.

1920 Data. 4912 Ayrshire Cows. Age Distribution. Type I Curve.



With respect to butter fat, a point of practical interest is the proportion of the whole number of cows which show an average percentage of butter fat during a lactation period of less than three per cent. In view of the present Regulations of the Ministry of Agriculture and Fisheries and of the Board of Agriculture for Scotland, respectively, it is clear that these poor cows should be eliminated from a herd. During a large portion of their lactation periods the milk of cows of this group reduces the quality of the bulked milk of the herds to which they belong. The writer has shown in another monograph* that the quality of bulked milk, as measured by the percentage of butter fat, depends very largely on the size of the herd whose milk is bulked for sale. The information supplied by the Scottish Milk Records Association enables dairymen to eliminate a bad cow at the end of its lactation period. The dairyman knows from the records the average percentage of butter fat obtained from periodic tests, the results being averaged at the end of

* "Variations in the Composition of Milk," H.M. Stationery Office, Edinburgh.

DIAGRAM XLIII.

1920 Data.

4912 Ayrshire Cows. Percentage Butter Fat Distribution. Type IV Curve.

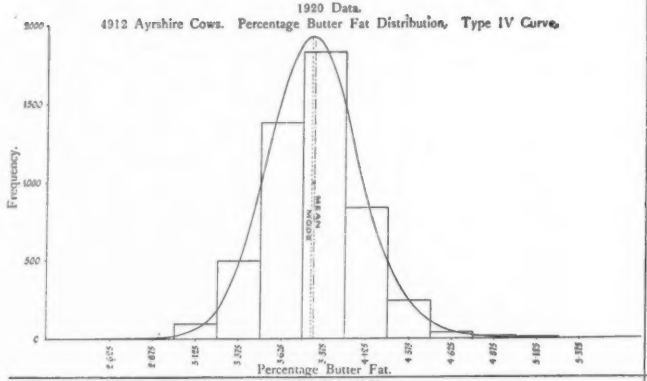


DIAGRAM XLIV.

1920 Data.

4912 Ayrshire Cows. Distribution of Yield of Butter Fat per Week. Type IV Curve.

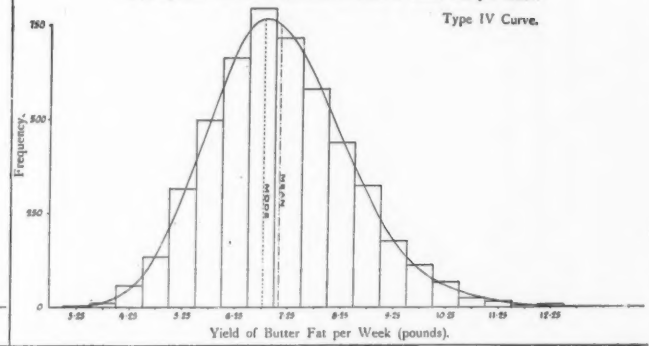


DIAGRAM XLV.

1920 Data.

4912 Ayrshire Cows. Weeks in Milk Distribution. Type IV Curve.

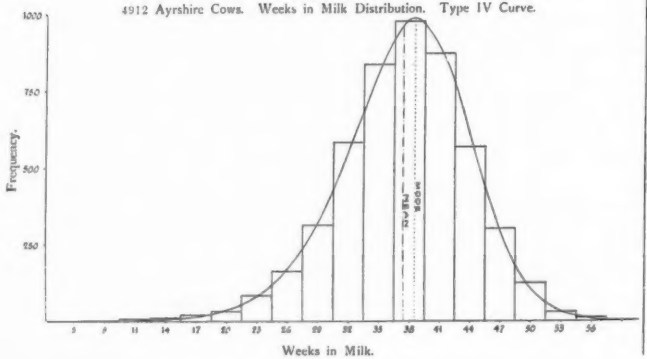
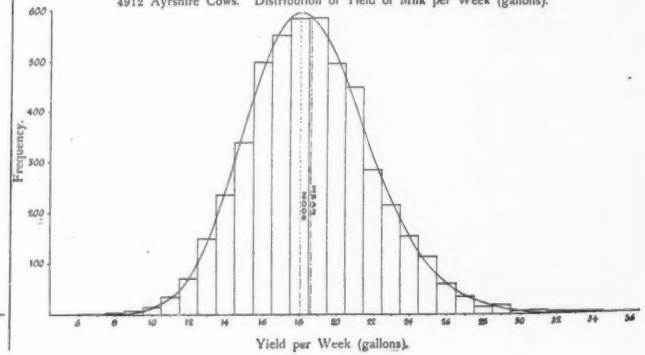


DIAGRAM XLVI.

1920 Data.

4912 Ayrshire Cows. Distribution of Yield of Milk per Week (gallons).



the lactation period. He can raise the standard of quality in his herd by the process of elimination. This elimination, however, must be carried out, giving due consideration to other factors such as breed and age of cow.

(*b*) *Analysis of Monier-Williams Data.*

Dr G. W. Monier-Williams has kindly supplied the writer with raw data giving the results of analyses of milk for percentages of butter fat in a large herd of cows —65 in number. The data consist of the percentages of butter fat found from samples of the morning and evening milk of these cows taken over a definite period. My statistical examination of the Monier-Williams data reveals the fact that the percentage of butter fat in morning milk is significantly less than the percentage of butter fat in evening milk. The average figure for morning milk is in the neighbourhood of 3·2 per cent., while the corresponding value for evening milk is in the neighbourhood of 4 per cent.

Since butter fat percentage is a function of age, it is clear that the fitting of frequency curves to butter fat has no physical meaning, the material being more or less heterogeneous. Monier-Williams* as a first approximation fitted normal curves to these distributions but it appeared to me on an examination of the data that the distributions were skew and not symmetrical. I therefore analysed the data to find the best fitting curves to the various distributions, in order merely to give the best possible description of the data. The best fitting curves were found mainly to be Type IV curves and not normal curves.

Out of thirteen series, one Type I curve was indicated and one normal curve, while the remaining eleven were Type IV curves. The table on the following page (Table XXX) shows the results of our analyses of the data supplied by Dr Monier-Williams. The best fitting curve of the series is No. 3 in the table—the morning milk of the complete herd for the whole period under review. Series No. 13—Dairy Company, Newcastle—is also a good fit. It will be seen that the fit is very bad in the cases of Nos. 10 and 11. Only in one case is the fit good for the Dairy Company's distributions, viz., No. 13. Several factors would affect the goodness of fit, including age of cow, stage of lactation period, length of interval between milkings and the degree of selection practised by the owners of the herd, and thus the bad fits are indications of the extent of the heterogeneity of the data. It will be seen that the coefficients of variation in the case of the Moundsmere Herd run from 14·9 to 19·8. In the case of Lanark group, the corresponding figure is approximately 16. In all these cases, the samples were individual cows. The remaining series were samples from bulked milk and it is natural to expect a smaller coefficient of variation in these cases. The values range from 8·5 to 12·4 (Diagrams XLVII—LIX). There does not appear from the data to be any significant difference between the milk from cows fed on winter rations and the milk from cows when they are grass fed. This statement refers only to the percentages of butter fat, as we have not examined the data for yield.

* "An Examination of Certain Milk Records," *Analyst*, June, 1920.

TABLE XXX.

Monier-Williams Data—Values of Frequency Constants and Nature of Curves fitted to the Distributions of Percentage Butter Fat.

Type IV Curves fitted in each case except for 9 *and* 12 *which were fitted to Type I and Normal Curves respectively.*

Source of Samples*	Morning or Evening Milk	N	Mean	Mode	Mean less Mode	Standard Deviation	Coefficient of Variation	β_1	β_2	κ_2	Observed Range	χ^2	P
Moundsmere Herd.													
1. Winter Rations	M.	1136	3·278 ± ·012	3·186	·092	·603—·009	18·4	·184	3·980	·104	1·4—5·8	22·5	·43
2. Grass Fed	M.	1277	3·170 ± ·012	3·128	·042	·627—·008	19·8	·066	4·941	·014	1·0—7·0	29·9	·12
3. Whole Period	M.	2413	3.221 ± ·008	3·164	·057	·618—·006	19·2	·102	4·530	·028	1·0—7·0	12·6	·97
4. Winter Rations	E.	1141	4·243 ± ·013	4·142	·101	·631—·008	14·9	·315	4·974	·088	1·8—7·2	26·6	·32
5. Grass Fed	E.	1271	4·156 ± ·013	3·992	·164	·708—·009	17·0	·488	4·718	·212	1·4—7·2	37·0	·07
6. Whole Period	E.	2412	4·197 ± ·009	4·068	·129	·674—·007	16·1	·389	4·812	·133	1·4—7·2	42·3	·03
Other Data.													
7. Lanark	M.	754	3·640 ± ·015	3·540	·100	·609—·011	16·7	·205	3·973	·123	1·8—6·2	26·3	·16
8. Lanark	E.	757	3·977 ± ·016	3·917	·060	·635—·011	16·0	·070	3·778	·040	1·8—6·2	24·7	·26
9. Dairy Co. A, Wilts United	—	5593	3·592 ± ·004	3·449	·143	·446—·003	12·4	·288	3·177	·456	2·2—6·0	35·7	·003
10. Dairy Co. B, Riley's, Hull	M.	4377	3·557 ± ·004	3·468	·089	·378—·003	10·6	·311	3·785	·397	2·4—5·4	76·3	very small
11. Dairy Co. B, Riley's, Hull	E.	2460	4·139 ± ·005	4·051	·087	·350—·003	8·5	·326	3·726	·556	3·0—5·8	58·7	very small
12. Dairy Co. C, Carrick's, Newcastle	M.	1499	3·321 ± ·006	3·292	·029	·317—·004	9·6	·033	3·040	1·388	2·2—4·6	22·9	·01
13. Dairy Co. C, Carrick's Newcastle	E.	1417	3·819 ± ·006	3·806	·014	·345—·004	9·0	·011	3·561	·952	2·6—5·0	6·2	·91

* 1—8 inclusive, are samples from individual cows; 9—13 inclusive, are samples of the mixed milk of several cows.

DIAGRAM XLVII.

Monier-Williams Data.
Morning Milk (Moundsmere) Winter Rations. Complete Herd
Percentage Butter Fat. Type IV Curve

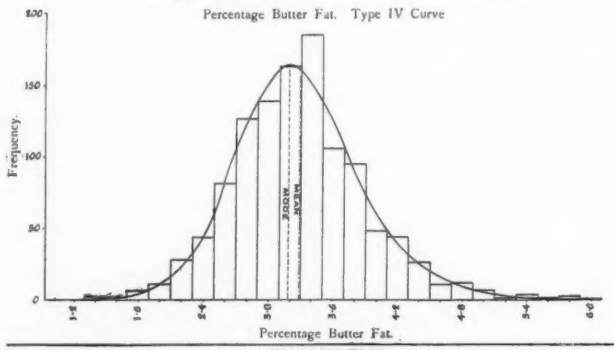


DIAGRAM XLVIII.

Monier-Williams' Data.
Morning Milk (Moundsmere). Grass Fed. Complete Herd.
Percentage Butter Fat. Type IV Curve.

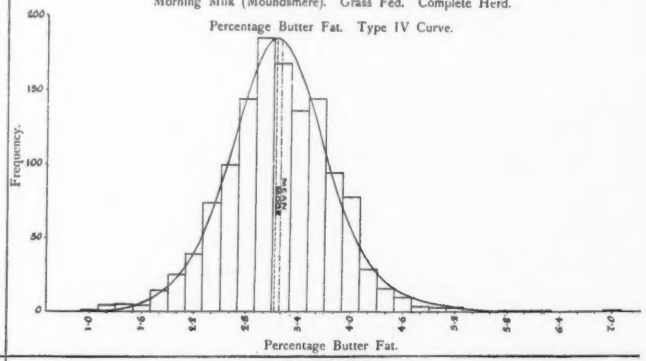


DIAGRAM XLIX.

Monier-Williams' Data.
Morning Milk (Moundsmere). Whole Period. Complete Herd.
Percentage Butter Fat. Type IV Curve.

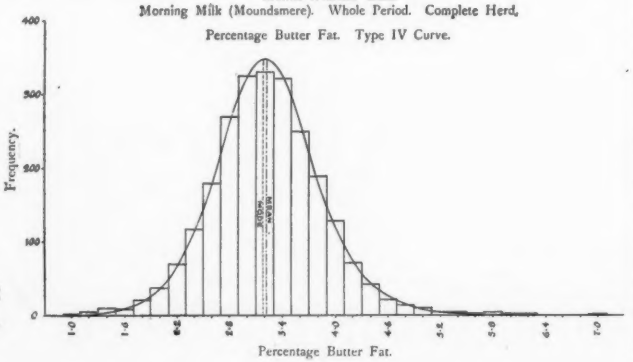


DIAGRAM L.

Monier-Williams' Data.
Evening Milk (Moundsmere). Winter Rations. Complete Herd.
Percentage Butter Fat. Type IV Curve.

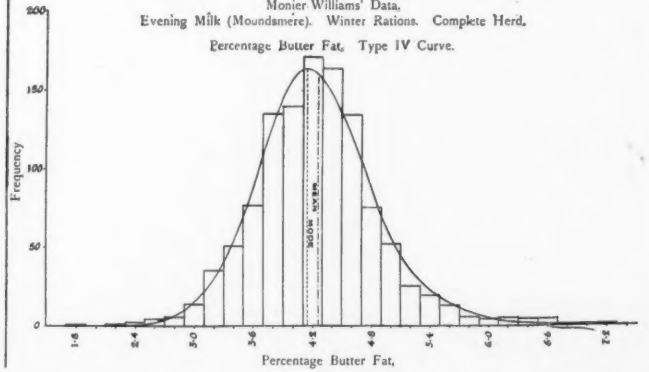


DIAGRAM LI.

Monier-Williams' Data.
Evening Milk (Moundsmere). Grass Fed. Complete Herd.
Percentage Butter Fat. Type IV Curve.

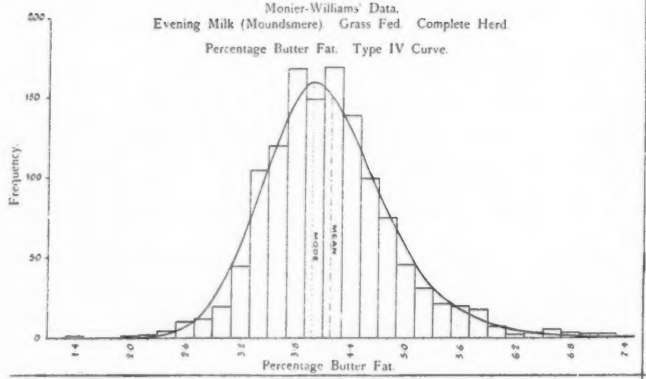


DIAGRAM LII.

Monier-Williams' Data.
Evening Milk (Moundsmere). Whole Period. Complete Herd.
Percentage Butter Fat. Type IV Curve.

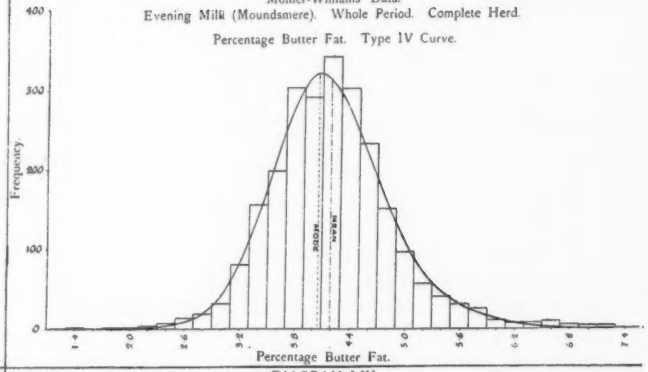


DIAGRAM LIII.

Monier-Williams Data.
Lanark Morning Milk.
Percentage Butter Fat. Type IV Curve.

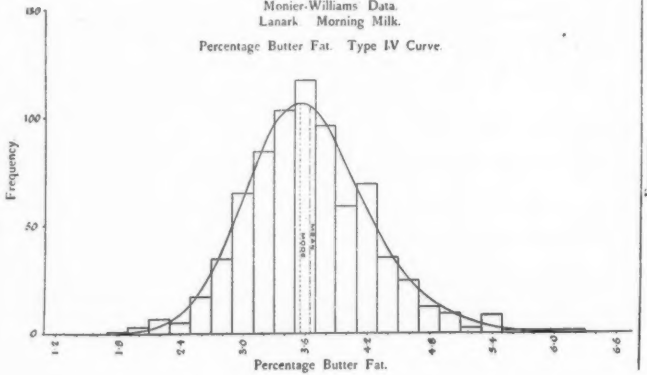


DIAGRAM LIV.

Monier-Williams' Data.
Lanark. Evening Milk.
Percentage Butter Fat. Type IV Curve.

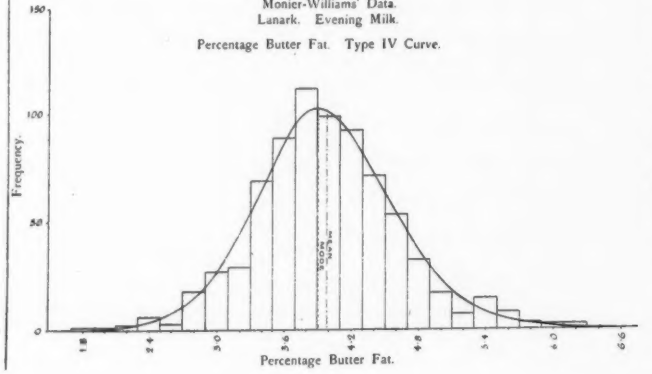


DIAGRAM LV.

Monier-Williams' Data.
Dairy Co. A. (Wilts Limited).

Percentage Butter Fat. Type I Curve.

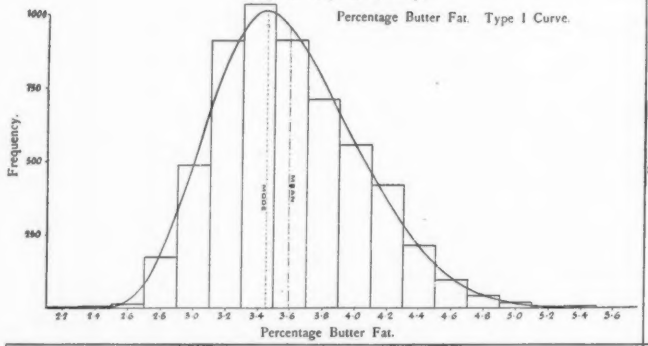


DIAGRAM LVI.

Monier-Williams' Data.
Dairy Co. B. (Rileys' Dairy, Hull). Morning.

Percentage Butter Fat. Type IV Curve.

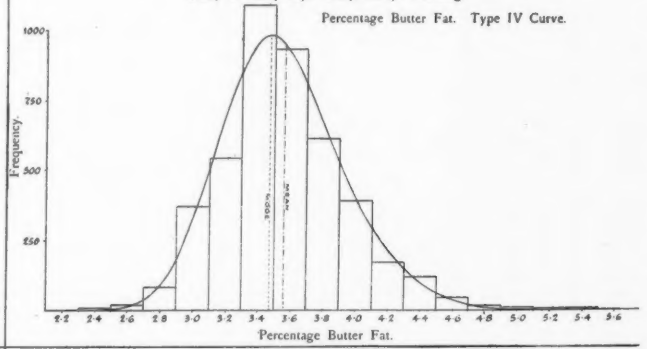


DIAGRAM LVII.

Monier-Williams' Data.
Dairy Co. B. Evening.

Percentage Butter Fat. Type IV Curve.

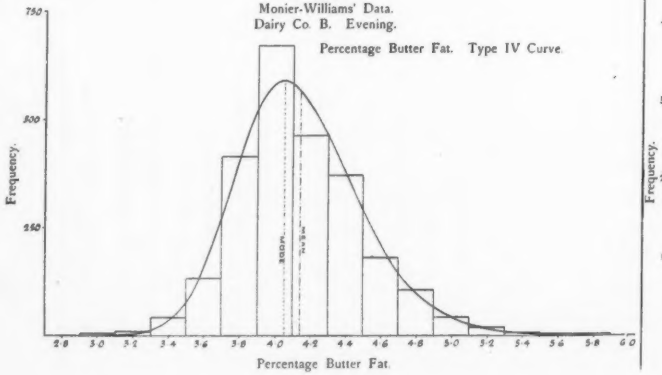
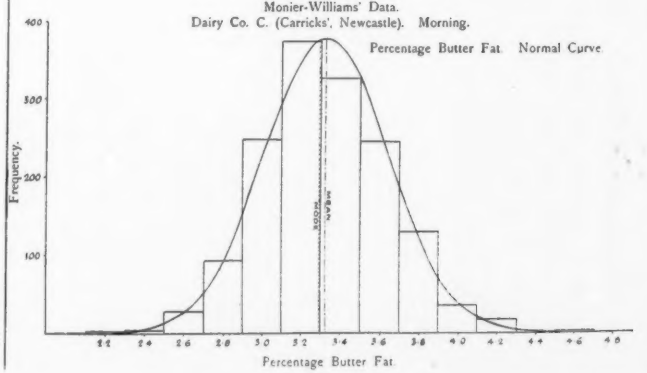


DIAGRAM LVIII.

Monier-Williams' Data.
Dairy Co. C. (Carricks', Newcastle). Morning.

Percentage Butter Fat. Normal Curve.



(c) *Distributions of Yield for each Year of Age.*

It will be noted for the 1911–12 data, the actual yields *for each lactation period* of each cow were analysed, and the tables and graphs show the nature of these yields. For the 1920 data, the *yields per week were* studied (Diagrams LX—LXIX). A reference to Table XXXI shows that in six cases Type IV curves fit the distributions, while in three cases Type I curves are the best fits. The 10 years' age

TABLE XXXI.

Milk Records—1920.

Character—Yield per Week												
Range	Class Centre	N	Type	μ_2	μ_3	μ_4	β_1	β_2	P	Mean	Mode	Skewness
From to												
2·5— 3·5	3	112	I	10·5550	11·5185	334·2782	·1128	3·0005	·9931	16·8839	16·2691	·1892
3·5— 4·5	4	1129	I	9·0745	8·0052	249·9587	·0858	3·0355	·3671	17·3570	16·8868	·1561
4·5— 5·5	5	1047	IV	10·4501	12·4201	370·6085	·1352	3·3937	·9628	18·3219	17·7910	·1642
5·5— 6·5	6	812	IV	11·2249	11·5414	422·3679	·0942	3·3522	·3776	18·9421	18·4883	·1355
6·5— 7·5	7	636	IV	10·6007	6·6423	358·9311	·0267	3·1941	·9541	19·1981	18·9561	·0743
7·5— 8·5	8	419	I	10·9327	8·8477	339·0737	·0599	2·8368	·6963	19·9451	19·4556	·1480
8·5— 9·5	9	276	IV	11·7635	21·4242	501·1432	·2820	3·6215	·7493	19·3913	18·5782	·2371
9·5—10·5	10	223	II	9·1223	1·0845	234·0643	·0015	2·8127	·8737	19·8341	19·7657	—
10·5—11·5	11	122	IV	17·1943	34·5236	1197·8335	·2345	4·0516	·8833	19·7951	19·0765	·1733
11·5—	12·86	136	IV	12·7080	17·5511	610·9823	·1501	3·7833	·9283	19·1324	18·6128	·1457

Mean $P = ·7786$

group was fitted with a Type II curve. In all cases these curves were found to give good fits for the respective distributions. The curves are all positively asymmetrical, the degree of skewness ranging from ·07 to ·19, values less than the degree of skewness for distributions of total yields for all lactation periods (1911–12). The mean weekly yield increases slowly with increasing age from 16·9 to 19·8 gallons for the 11 years' age group. Cows over 11·5 years of age were studied together. The mean yield for this group is slightly less than the corresponding figure for the 11 years' age group. The modal values also increase with increasing age from 16·3 to 19·1 gallons for 11 years of age. A reference to Diagram LXX shows the extent of the regression of array standard deviations of yield on age. It will be seen that the array variabilities increase from about 3·1 at 3 years of age to 3·6 gallons at 12 years of age. *Older cows are therefore more variable in their milk yield than young cows**. This is found to be generally the case, as will be seen from the next subsection, where an analysis of the 1908–9 data has been carried out, with respect to yield and its variability.

* A scedastic curve fitted to *fat percentage* standard deviations on age reveals the fact that older cows are slightly less variable in fat percentages than younger cows.

| | | |

(3) *Analysis of* 1908–9 *Data* (*all Ayrshires and Fenwick District*) *and the Work of Pearl and Vigor.*

With regard to Pearl's analysis* of the Scottish Records for 1908–9 in which he finds that, for 6935 Ayrshire cows, the weighted mean standard deviation of the arrays of weekly yield on age is 2·806 gallons, the comparable figures from Vigor's analysis for the Fenwick district† of the Scottish Milk Records Association (586 cows) would be the corresponding weighted mean standard deviation of the arrays of weekly yield on age. From Vigor's table we find the value of that constant to be $\bar{\sigma}_{Q_A}=3{\cdot}415$, and not the figure 4·0704, which Pearl uses and which is the standard deviation of the whole distribution of weekly yield for Fenwick cows of all ages. It is well known that where there is a correlation between two variates, the mean array standard deviation is less than the corresponding standard deviation of the whole distribution. Pearl, however, correctly contrasts the figure 4·070$\frac{4}{}$ (Fenwick 1909) with the standard deviation of the whole distribution of weekly yield for 6935 Ayrshire cows of all ages (1908–9), namely, 3·329 (as found in his Table V, p. 300, *Journal of Agricultural Research*, Vol. XVII, September 1919). The table below (Table XXXII) shows the respective values of $\bar{\sigma}_{Q_A}=S(n_A\sigma_{Q_A})/N$ and σ_Q; the

TABLE XXXII.

A.

Area	Weighted mean of Array Standard Deviations $=\bar{\sigma}_{Q_A}$	Standard Deviation of Yield $=\sigma_Q$	Mean Weekly Yield $=\bar{Q}$
Ayrshires 1908–9	2·806	3·329	16·49 gallons
Fenwick 1909	3·415	4·070	16·83 ,,
Ayrshires 1920	3·244	3·370	18·61 ,,
Fenwick 1920	3·376	3·538	19·37 ,,

B. *Percentage Increase in Fenwick over all Ayrshires.*

In $\bar{\sigma}_{Q_A}$		In σ_Q	
1908–9	21·70	1908–9	22·27
1920	4·07	1920	4·75

C. *Percentage Decrease of* $(\sigma_Q-\bar{\sigma}_{Q_A})$ *on* σ_Q.

Ayrshires 1908–9	15·71
Fenwick 1909	16·01
Ayrshires 1920	3·72
Fenwick 1920	4·58

* There is an error in Pearl's figure giving the mean weekly yield. His figure 15·991 should be 16·491 gallons weekly yield. The error is due to his taking the lower limit of his sub-range of yield (Table V, page 300) instead of the centre of his sub-range.

† *Supplement to Journal of Board of Agriculture and Fisheries*, No. 11, by H. D. Vigor, August 1913.

DIAGRAM LXIII.

1920 Records.

Yield of Milk per Week for 6 Years of Age. Type IV Curve.

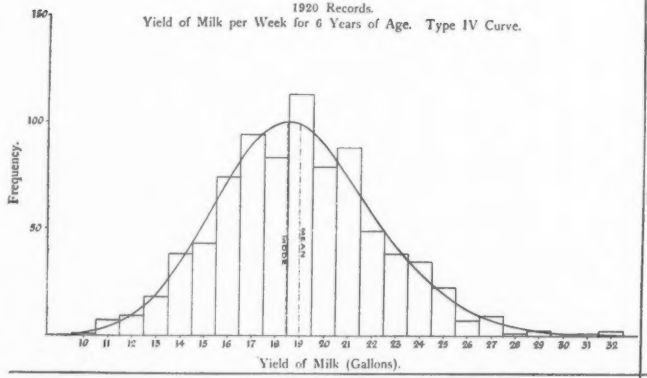


DIAGRAM LXIV.

1920 Records.

Yield of Milk per Week for 7 Years of Age. Type IV Curve.

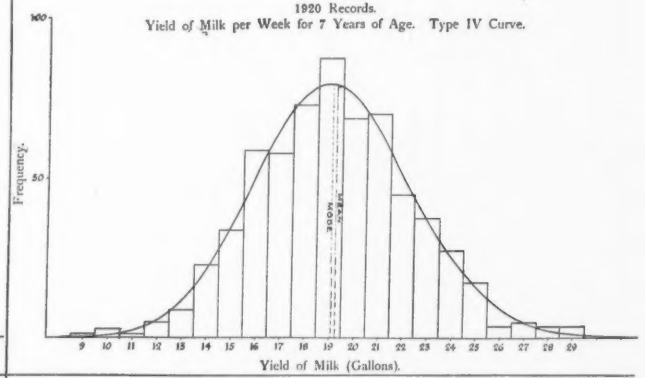


DIAGRAM LXV.

1920 Records.

Yield of Milk per Week for 8 Years of Age. Type I Curve.

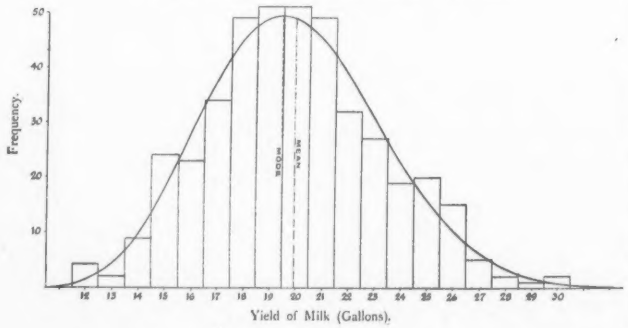
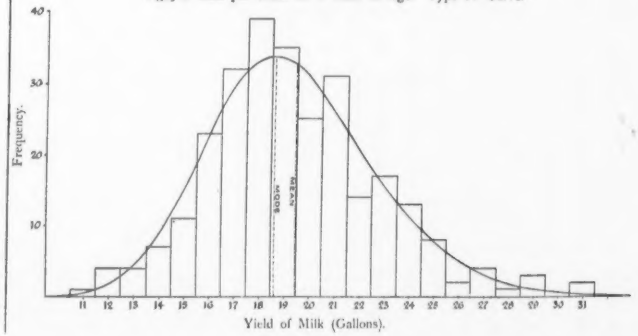


DIAGRAM LXVI.

1920 Records.

Yield of Milk per Week for 9 Years of Age. Type IV Curve.



percentage increases in $\bar{\sigma}_{Q_A}$ and σ_Q in Fenwick over all Ayrshires; and the percentage decreases of $(\sigma_Q - \bar{\sigma}_{Q_A})$ on σ_Q for all Ayrshires and for Fenwick district.

The percentage reductions in $(\sigma_Q - \bar{\sigma}_{Q_A})$ for 1908–9 are practically the same for both areas (16 per cent.), while the corresponding figures for 1920 are approximately one-fourth the 1908–9 figures. There is therefore a decrease in the correlations between age and yield in 1920, when compared with the values in 1908–9. The percentage increases in Fenwick (1909) over all Ayrshires (1909) are practically the same for $\bar{\sigma}_{Q_A}$ and σ_Q. The corresponding increases for 1920 are approximately one-fifth the figures for 1909. We find for Ayrshire cows (Pearl's Tables, 1908–9) $\eta_{QA} = \cdot 5330$ and for Fenwick district (Vigor's Tables, 1909) $\eta_{QA} = \cdot 5290$. Thus the correlations as measured by the correlation ratio are practically the same for both groups in 1908–9. In the Report for 1909 of the then "Ayrshire Cattle Milk Records Committee," it is stated (p. 13) that "in no district is the value attaching to milking pedigree so well appreciated and acted on as in the Fenwick district." Greater proportions of good milkers in this district than the proportions throughout the remaining districts would clearly increase the value of σ_Q in Fenwick over σ_Q in all Ayrshires, and it would also increase the value of $\bar{\sigma}_{Q_A}$ since there is no reason to suppose that particular ages would be selected by Fenwick dairymen. Only 8·9 per cent. of all Ayrshires gave yields above 20 gallons, while 15·5 per cent. of Fenwick cows gave yields above 20. The proportion of poor yielders of Fenwick (10 gallons and below) was 5·8 as against 4·1 per cent. the corresponding figure for all Ayrshires. The Fenwick results for 1920, however, resemble the all Ayrshire results for 1920, both with respect to σ_Q and $\bar{\sigma}_{Q_A}$. Further, the values of η_{QA} for 1920 are ·2568 for all Ayrshires and ·2196 for Fenwick. *It is thus clear that weekly yield was more a function of age in both groups during* 1908 *to* 1909 *than it was in* 1920. The figures for 1911–12 are not comparable with those of 1908, 1909 or 1920, because for 1911 and 1912 the analysis was performed on the *total* yield and not on the *weekly* yield.

The difference between σ_Q and $\bar{\sigma}_{Q_A}$ is not wholly accounted for by the change in mean weekly yield with age as Pearl suggests. Another factor operates in this direction, though to a much less degree, viz. the variability in the array standard deviations. In Pearson's memoir on Skew Correlation* we have

$$\sigma^2_Q = \sigma^2_{a_Q} + \sigma^2_{m_Q},$$

where $\sigma^2_{a_Q} = S(n_A \sigma^2_{Q_A})/N$ and $\sigma^2_{m_Q} = S\{n_A(\bar{Q}_A - \bar{Q})^2\}/N$.

Now consider the variability of the array standard deviations round the mean array standard deviation. It is easy to see that

$$S\{n_A(\sigma_{Q_A} - \bar{\sigma}_{Q_A})^2\}/N = S(n_A \sigma^2_{Q_A})/N - \left\{\frac{S\{n_A \sigma_{Q_A}\}}{N}\right\}^2,$$

or

$$\Sigma^2 \sigma_{Q_A} = \sigma^2_{a_Q} - (\bar{\sigma}_{Q_A})^2.$$

Hence

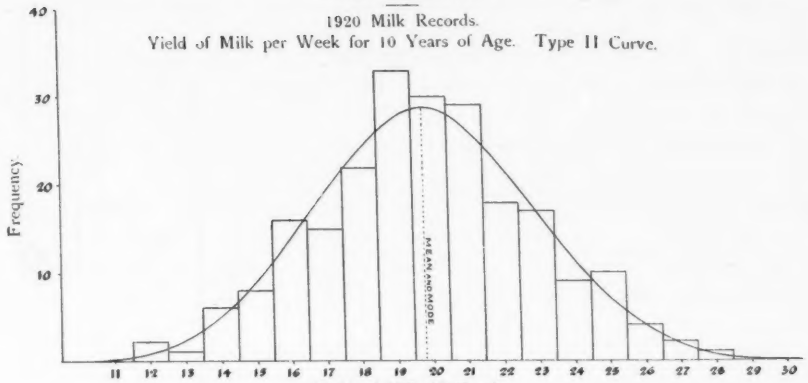
$$\sigma^2_Q = \Sigma^2 \sigma_{Q_A} + (\bar{\sigma}_{Q_A})^2 + \sigma^2_{m_Q}.$$

* *Skew Correlation and Non-linear Regression*, Biometric Series, I.

DIAGRAM LXVII.

1920 Milk Records.

Yield of Milk per Week for 10 Years of Age. Type II Curve.

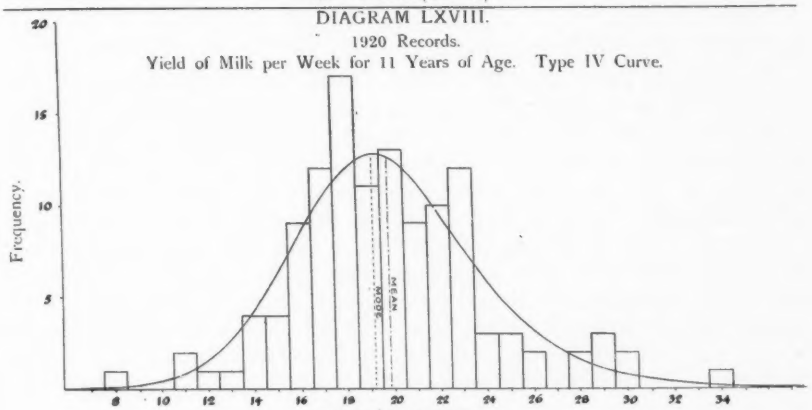


Yield of Milk (Gallons).

DIAGRAM LXVIII.

1920 Records.

Yield of Milk per Week for 11 Years of Age. Type IV Curve.

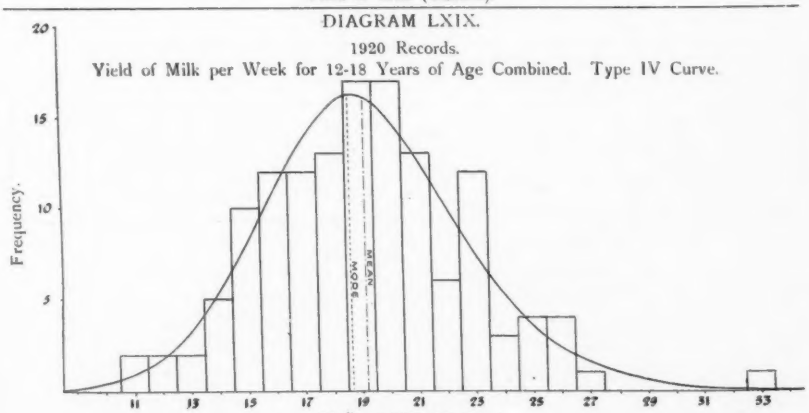


Yield of Milk (Gallons).

DIAGRAM LXIX.

1920 Records.

Yield of Milk per Week for 12-18 Years of Age Combined. Type IV Curve.



Yield of Milk (Gallons).

The following table (Table XXXIII) shows the values of the various constants.

TABLE XXXIII.

Area	(*a*) $\Sigma^2\sigma'_{Q_A}$	(*b*) $\Sigma^2\sigma_{Q_A}$	(*c*) $(\bar{\sigma}_{Q_A})^2$	(*d*) $\sigma^2_{m_Q}$	(*e*) σ^2_Q	
Fenwick 1909	·0535	·30	11·66	4·64	16·60	
All Ayrshires 1908–9	·0403	·05	7·87	3·18	11·10	Actual values
Fenwick 1920	·0008	·51	11·40	·61	12·52	
All Ayrshires 1920	·0246	·04	10·53	·80	11·37	
Fenwick 1909	—	1·81	70·24	27·95	100	
All Ayrshires 1908–9	—	·45	70·90	28·65	100	Per cent. of σ^2_Q
Fenwick 1920	—	4·07	91·06	4·87	100	
All Ayrshires 1920	—	·35	92 61	7·04	100	

The following facts emerge from Tables XXXIII and XXXIV, namely: (1) In 1909 and 1920, particularly in 1909, the variances of the *array standard deviations* for Fenwick are high when compared with the corresponding figures for all Ayrshires (column (*b*) of XXXIII). In other words, the cows of the Fenwick district show both in 1909 and 1920 greater variability in weekly yield as the cows get older than Ayrshires in general do. (2) On the other hand, there are decreases in the variances of the *array means* of both areas in 1920 when compared with 1909 (column (*d*) of XXXIII). This, as has been already pointed out, indicates a reduction in the correlations in 1920 when compared with the corresponding 1909 correlations. (3) There is a distinct increase in the square of the weighted *mean of array standard deviations* of yield for all Ayrshires (1920) when compared with the corresponding figure for 1909 (column (*c*) of XXXIII). The values for Fenwick

TABLE XXXIV.

Area and Date	$\eta_{Q.A}$	$\eta'_{Q.A}$	$\eta''_{Q.A}$	ϕ_α	ϕ_β	$(\phi_\alpha+\phi_\beta)$
Fenwick 1909	·5290	·5456	·6418	6·07	93·93	100
All Ayrshires 1908–9	·5330	·5394	·6610	1·55	98·45	100
Fenwick 1920	·2196	·2991	·7076	45·53	54·47	100
All Ayrshires 1920	·2658	·2720	·6711	4·82	95·18	100

$\eta_{Q.A}$ = Correlation Ratios found from data.

$\eta'_{Q.A}$ = Correlation Ratios on assumption of homoscedasticity.

$\eta''_{Q.A}$ = Correlation Ratios on assumption of homoscedasticity, and value of 2·5 gallons for array standard deviations.

$\phi_\alpha = (100\Sigma^2\sigma_{Q_A})/\{\sigma^2_Q - (\bar{\sigma}_{Q_A})^2\}$.

$\phi_\beta = (100\sigma^2_{m_Q})/\{\sigma^2_Q - (\bar{\sigma}_{Q_A})^2\}$.

DIAGRAM LXX.

Regression of Standard Deviations of Arrays of Yield on Age.

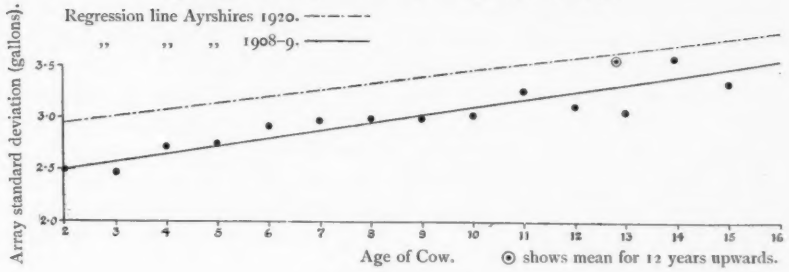


DIAGRAM LXXI.

Regression of Standard Deviations of Arrays of Yield on Age.

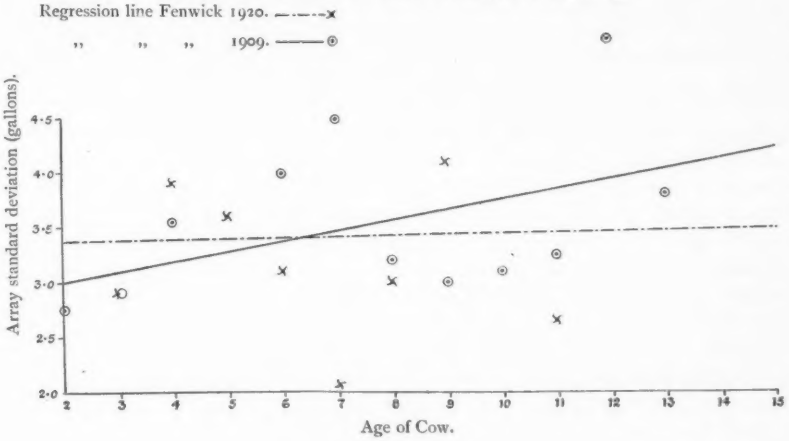
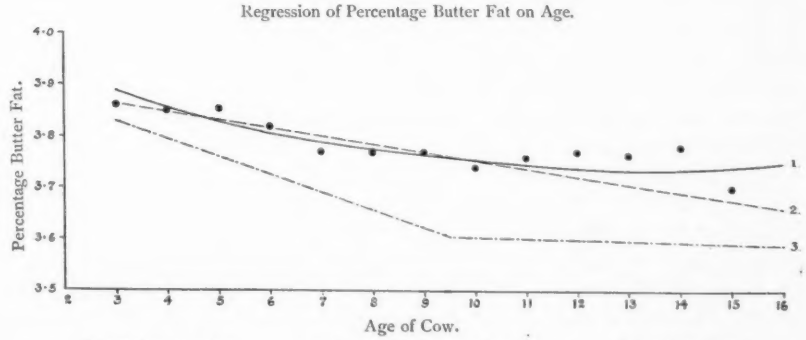


DIAGRAM LXXII.

Regression of Percentage Butter Fat on Age.



1. Parabola, 1920 data. 2. Straight line, 1920 data. 3. 1908–9 data (Pearl).

are practically the same for 1909 and 1920. (4) There is a decrease in the *general variance* of yield in Fenwick in 1920 when compared with the figure for 1909 (column (*e*) of XXXIII). The general variances of yield for all Ayrshires are practically the same in 1909 and 1920. (5) While the correlation ratios η_{QA} have practically the same value for both groups in 1920, the reasons for their smallness when compared with the corresponding values of 1909, are different for the two groups (see Table XXXIV). (6) The different causes of the lower correlations between age and yield in 1920, when compared with 1909, are (*a*) the higher squared mean of the array standard deviations relative to the general variance in both cases, and (*b*) the higher variance of the array standard deviations relative to general variance in Fenwick (1920). This means either an increase in the values of the array standard deviations with age, or a greater random scattering of individual values round the mean array standard deviations. A reference to Diagram LXXI shows that the scedastic line for Fenwick 1920 is practically parallel to the *A* axis and therefore theoretically little or no increase in the variability of the array standard deviations with age is indicated for Fenwick 1920. Table XXXIII, column (*a*), shows that $\Sigma^2\sigma'_{Q_A} = \cdot0008$ (where σ'_{Q_A} is the theoretical value on the assumption of linearity) as against $\Sigma^2\sigma_{Q_A} = \cdot51$. Diagram LXXI also shows the observed values of the array standard deviations for Fenwick 1909 to vary widely and irregularly round the linear scedastic line, a line showing that older cows are more variable than younger cows. These results may be due to smallness of sample, and possibly special selection in Fenwick of cows below 6 or 7 years of age, notably consistent and therefore less variable in yield than usual. (*c*) On the other hand, all Ayrshires, both in 1909 and 1920, tend to have lesser values for the array standard deviations for each year of age than Fenwick 1909 and 1920. A reference to Diagram LXX shows the extent of the regressions of array standard deviations of yield on age of 6935 Ayrshires 1908–9 and of 4912 in 1920. It must be concluded that the general population of Ayrshire cows (both 1908–9 and 1920) show increasing variability of yield with age. The increase is continuous and therefore we have here a case of *nomic* heteroscedasticity. It will be noted that the variability of yield with age is uniformly greater for 1920 Ayrshires, when compared with 1908–9 Ayrshires.

It is of interest to note what would be the effect of a selection designed to produce homoscedasticity. If the homoscedastic values of the array standard deviations had been equal to 2·806 gallons (Ayrshires 1908–9) and 3·415 gallons (Fenwick 1909) respectively, the values of η'_{QA} would have been ·5394 and ·5456 (see Table XXXIV).

Had the homoscedastic value of the array standard deviations been 2·5 gallons (the value found for cows 2½ years old) the corresponding values of η''_{QA} would range from ·64 to ·71 for both groups in 1909 and 1920. *We thus see that to produce a high correlation between quantity and age it would be necessary to have a low homoscedastic value, or low heteroscedastic values, for the variabilities of the arrays. This would mean a drastic elimination of poor yielders, for it would*

frustrate the object of milk recording to attempt to reduce variability by cutting off the high yielders. There has been a more drastic cutting off of poor yielders in Fenwick 1920, than in 1909 ($\sigma^2_Q = 16{\cdot}6$ in 1909 and $= 12{\cdot}52$ in 1920), but it has not been evenly applied to all ages ($\Sigma^2_{\sigma_{Q_A}} = {\cdot}51$ for 1920, and see Diagram LXXI for observed values of σ_{Q_A}).

Indeed, while the squared means of the array standard deviations for Fenwick were practically the same for 1909 and 1920 (11·66 and 11·40), the remaining contributions to the variance of the general distribution of yield in 1920 were nearly equally divided between the variance of the array standard deviations and the variance of the array means (0·51 and 0·61 respectively) while in 1909 about 94 per cent. was contributed by the variance of the array means and only 6 per cent. by the variance of the array standard deviations (see Table XXXIV values of ϕ_α and ϕ_β). *Thus the cause of the reduction of the correlation between age and yield in Fenwick from* 0·5290 *in* 1909 *to* 0·2196 *in* 1920 *was the higher value in* 1920 *of the ratio* $(\Sigma^2_{\sigma_{Q_A}} + (\bar{\sigma}_{Q_A})^2)/\sigma^2_Q$, *namely,* 0·95 *when compared with the corresponding value in* 1909, *namely* 0·72. In other words, the squared mean of the array standard deviations approached more nearly to the general variance of yield in 1920, than it did in 1909. This was accompanied by a greater variability in the array standard deviations in Fenwick in 1920 when compared with the corresponding figure for 1909, thus producing a fall in the value of η^2* (and therefore of η) in 1920.

The problem for the practical producer is whether or not he can reduce the greater variability of cows above 7 years of age by selection. If the least variable in yield of good cows under seven years of age were selected for milk production and the others rejected, this procedure would bring about at a later date more consistent yields of good cows above 7 years of age and would reduce their variabilities below their present values. The annual records of total yields of cows under 7 years of age provide the necessary data enabling the milk producer to select cows with the most consistent weekly yields over, say, 4 years. The data on which to base selection would thus be the *average* weekly yield,—because of the varying lengths of lactation periods. Only cows therefore, with completed lactation periods of a standard duration or over to be agreed upon, should form the basis of selection. The least variable over a period of years in average weekly yield of good cows should be retained for milk production for succeeding lactation periods.

The general result of this section of the investigation shows that there is an increase of yield with age of cow and also that yield gets more variable with age. The correlations between the factors are not constant in time. Different values of the constants are found for the various years indicating selective action by the milk producer.

* $\eta^2 = \left[1 - \frac{\{\Sigma^2_{\sigma_{Q_A}} + (\bar{\sigma}_{Q_A})^2\}}{\sigma^2_Q}\right]$

(4) *Interdependence of various Milk Factors.*

(*a*) *Correlation Ratios and Coefficients,* 1920 *Data.*

The correlation ratios and coefficients for eight pairs of characters were determined together with the divergence from linearity. The undernoted table (Table XXXV) shows the results.

TABLE XXXV.

Correlation Ratios and Coefficients.

Pairs of characters studied	N	η	r	$\frac{\sqrt{N}}{\cdot 67449} \times \frac{1}{2}\sqrt{\eta^2 - r^2}$
Age on Percentage Butter Fat	4912	·1317	− ·1247	2·14
Percentage Butter Fat on Age	4912	·1445	− ·1247	3·78
Age on Length of Lactation Period	4912	·0842	− ·0095	4·35
Length of Lactation Period on Age	4912	·0489	− ·0095	2·49
Age on Yield of Butter Fat per week	4912	·1719	·1648	2·49
Yield of Butter Fat per week on Age	4912	·2125	·1648	6·97
Age on Yield of Milk per week	4912	·2419	·2192	5·33
Yield of Milk per week on Age	4912	·2658	·2192	7·80
Length of Lactation Period on Percentage Butter Fat	4912	·1255	·0745	5·25
Percentage Butter Fat on Length of Lactation Period	4912	·1156	·0745	4·59
Length of Lactation Period on Yield of Milk per week	4912	·11[illegible]	− ·0314	5·67
Yield of Milk per week on Length of Lactation Period	4912	·1001	− ·0314	4·32
Length of Lactation Period on Yield of Butter Fat per week	4912	·0799	− ·0006	4·16
Yield of Butter Fat per week on Length of Lactation Period	4912	·1003	− ·0006	5·22
Yield of Milk per week on Percentage Butter Fat	4912	·1429	− ·1366	2·14
Percentage Butter Fat on Yield of Milk per week	4912	·1524	− ·1366	3·49

The correlations are all relatively low, the highest being the relationship between yield per week and age. It is seen that the relationships can be classified in the following manner:

Possibly linear.

Age and Percentage of Butter Fat.
Age and Yield of Butter Fat per week.
Length of Lactation Period and Age.
Yield of Milk per week and Percentage Butter Fat.

Non-Linear.

Percentage Butter Fat on Yield of Milk per week.
Percentage Butter Fat on Age.
Length of Lactation Period on Yield of Butter Fat per week.
Age on Length of Lactation Period.
Percentage Butter Fat on Length of Lactation Period.

Yield of Milk per week on Length of Lactation Period.
Yield of Butter Fat per week on Length of Lactation Period.
Length of Lactation Period on Percentage Butter Fat.
Age on Yield of Milk per week.
Length of Lactation Period on Yield of Milk per week.

Markedly non-linear.

Yield of Butter Fat per week on Age.
Yield of Milk per week on Age.

(*b*) *Regression Curves.*

Regression curves have been fitted for (1) Percentage Butter Fat on Age, (2) Milk Yield per week on Age, (3) Yield of Butter Fat per week on Age, and (4) Length of Lactation Period on Age.

(1) It is clear from Diagram LXXII that the average percentage of butter fat given by a cow during a lactation period gradually falls with age from about 3·9 per cent. at 3 years of age to about 3·75 per cent. at 10 years of age. After 10 years, the percentage practically remains constant up to 16 years of age. If the average percentage fell proportionally with age, the value at 16 years would be approximately 3·65 per cent. The results of Pearl's work on the 1908–9 data are shown on the same diagram. It will be seen that, for the 1908–9 data, the average percentage fell from about 3·85 per cent. at 3 years of age to about 3·6 per cent. at 10 years of age, after which the percentage remained constant until 16 years of age. It is clear from these results that there has been a slight but significant rise in the average proportion of butter fat in 1920 when compared with 1908–9.

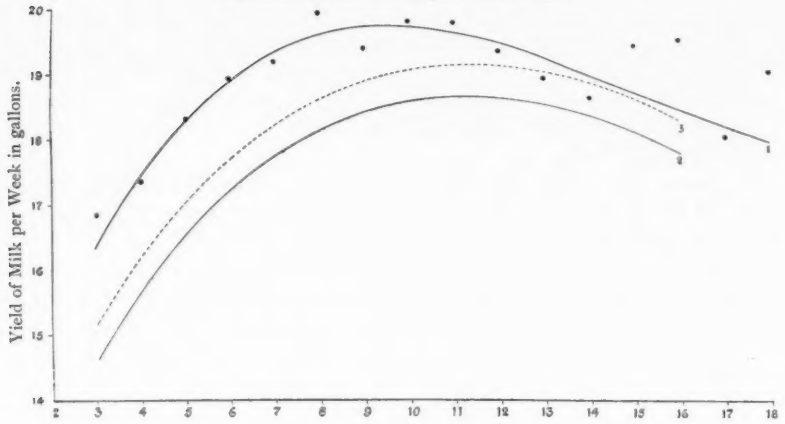
(2) It will be seen, on examining Diagram LXXIII (see (1) on diagram), that milk yield per week rose from about 16½ gallons for three year old cows to about 19¾ gallons for ten year old cows, after which the average milk yield per week gradually fell to about 18 gallons for cows of 18 years of age. A cubic was fitted to the data in this case. Pearl's results for 1908–9 are shown by the continuous line (2). This line, however, cannot be compared with the 1920 result, for the reason that a different method of estimating the total yield of milk per cow was adopted in the year 1916 (see Section (6) below on Comparison of Private with Association Records). The effect of this new method has been to add about half a gallon per week to the estimated yield. This average figure must therefore be added to the results for 1908–9 in order to make the two sets of figures reasonably comparable (see (3) on Diagram LXXIII).

(3) It will be seen from Diagram LXXIV that the average total yield of butter fat per week also rises with age. Three year old cows give about 6½ pounds and the yield of butter fat per week rises with age until about 10 years, when the quantity is about 7½ pounds. After that there is a progressive fall until about 16 years of age, when the average quantity per week is about 6·9 pounds. In this case a cubic has been fitted to the data.

(4) An inspection of Diagram LXXV will show that there is practically no relationship between length of lactation period and age of cow. The regression is linear and there is a slight fall which is scarcely significant.

DIAGRAM LXXIII.

Regression of Yield of Milk per Week on Age.



1. Cubic fitted to 1920 data. 2. Pearl, 1908–9 data. 3. 1908–9 adjusted.

DIAGRAM LXXIV.

Regression of Yield of Butter Fat per Week on Age.

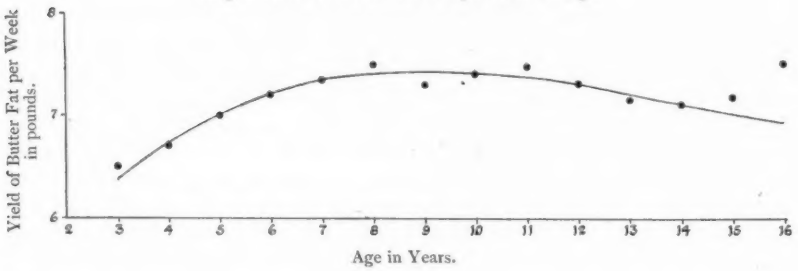
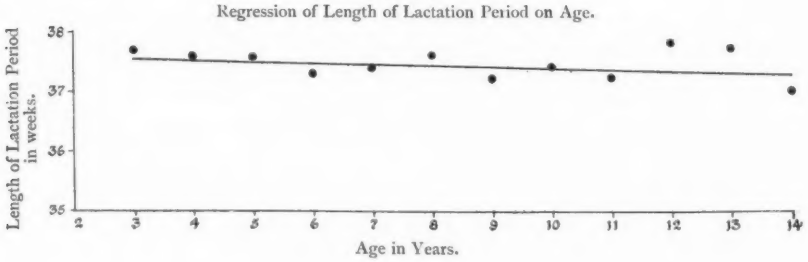


DIAGRAM LXXV.

Regression of Length of Lactation Period on Age.



(5) *Relation of Milk Factors to Date of Calving.*

At the suggestion of the Research Committee, the composition of milk was considered from the point of view of date of calving. Tables XLIII to XLVII of Appendix show the distributions of each character for cows calving in each month of the year. It was suggested that the data be studied in three four-monthly groups, A, B, and C (see tables below). It will be seen from Table XXXVI that the differences between the three groups in mean percentage of butter fat, mean yield of butter fat per week, and mean yield of milk per week are insignificant. The mean age of cows in Group C is, however, less than the mean ages of Groups A and B. On the other hand, the cows of Group C have a longer average lactation period than the other two groups, with the result that the mean yield for a lactation period is higher for this group than for the other two groups. These figures confirm the results found for 1912, as shown in the table.

TABLE XXXVI.

Means for Four-monthly Periods.

Character	1920			1912		
	Group A	Group B	Group C	Group A	Group B	Group C
	$N=3159$	$N=1211$	$N=542$	$N=3872$	$N=1986$	$N=790$
Age in years	6·33	6·26	5·54	6·37	6·15	5·41
Length of Lactation Period in weeks	37·31	36·35	40·71	37·86	34·54	42·41
Percentage of Butter Fat	3·80	3·82	3·86	3·72	3·77	3·75
Yield of Butter Fat per week in pounds	7·14	6·97	7·07	—	—	—
Yield of Milk per week in gallons	18·80	18·25	18·31	17·38	17·78	17·38
Total Yield for Lactation Period in gallons	701·37	663·31	745·34	658	614	737

Group A: December, January, February, March.
Group B: April, May, June, July.
Group C: August, September, October, November.

(6) *Analysis of Data for* 14,416 *Ayrshire Cows.*

In order to make a comparison between the results for the 4912 cows, with completed lactation periods and records of a second calf within 60 weeks of the previous calving, and the total number of Ayrshire cows for 1920, an analysis was conducted on the records of the 14,416 cows.

(*a*) *Means and Standard Deviations.*

The means and standard deviations for the characters observed are shown in the following table (Table XXXVII), together with the results for the 4912 cows.

TABLE XXXVII.

Means and Standard Deviations.

	14,416		4912	
	Mean	S. D.	Mean	S. D.
Age in years	6·399	2·36	6·223	2·21
Length of Lactation Period in weeks	34·925	10·50	37·447	6·37
Percentage Butter Fat	3·818	·29	3·814	·27
Yield of Butter Fat per week in pounds	7·286	1·57	7·091	1·32
Yield of Milk per week in gallons	19·085	3·89	18·610	3·37

It will be seen that the mean age for 14,416 cows is practically the same as the mean age for the selected group of 4912 cows. As would naturally be expected, the mean lactation period for all cows is shorter and more variable than for the selected group owing to the fact that some of the lactation periods were incomplete. There is a slightly greater average yield of butter fat per week and a slightly greater yield of milk per week for all cows when compared with the corresponding figures for the 4912 cows. Thus, little error would be introduced in taking the mean values for all cows to represent the mean values of the selected group with the record of a second calf within 60 weeks and with lactation periods completed during the year 1920. The slightly higher average yield of milk per week and the slightly higher average yield of butter fat per week are easily seen to be due to the fact that a large number of cows are included with incomplete lactation periods, and in consequence have yields recorded at periods where the maximum was obtained.

(*b*) *Correlations.*

The following table (Table XXXVIII) shows the values of the correlation ratios and coefficients for four pairs of characters for cows of all ages and for ages 5 to 7 years separately.

It will be noted from Table XXXVIII that there is an increase in the negative correlation between yield of milk per week and length of lactation period for cows all the same age when compared with the value for cows of all ages. The relationship of yield of butter fat to length of lactation period is not sensibly affected by age. The other pairs of characters are not appreciably affected by age, but it is to be noted that the correlations are in these cases quite small in magnitude.

(*c*) *Regressions.*

(1) *Percentage Butter Fat on Age.* A study of Diagram LXXVI shows that the average percentage of butter fat fell from 3·9 per cent. for three year old cows to about 3·72 per cent. for the oldest cows. That is to say, the fall was greater in the case of 14,416 cows than in the case of the selected group of 4912.

TABLE XXXVIII.

Correlation Coefficients and Ratios.

14,416 *Cows.*

Characters	All Ages			4 Years			5 Years			6 Years			7 Years		
	N	*r*	η	*N*	*r*	η	*N*	*r*	η	*N*	*r*	η	*N*	*r*	η
Yield of Milk per week on Length of Lactation Period	14,416	−·2993	·3718	3179	−·3273	·4078	2820	−·3441	·3994	2233	−·3478	·3940	1810	−·3284	·3962
Length of Lactation Period on Yield of Milk per week	14,416	−·2993	·3411	3179	−·3273	·4379	2820	−·3441	·4346	2233	−·3478	·4293	1810	−·3284	·4213
Percentage Butter Fat on Length of Lactation Period	14,416	−·0650	·1750	3179	−·0517	·1771	2820	−·0664	·1833	2233	−·0567	·1937	1810	−·0243	·1517
Length of Lactation Period on Percentage Butter Fat	14,416	−·0650	·1683	3179	−·0517	·1979	2820	−·0664	·1971	2233	−·0567	·2239	1810	−·0243	·1425
Yield of Butter Fat per week on Length of Lactation Period	14,416	−·3567	·3820	3179	−·3403	·4524	2820	−·3640	·4458	2233	−·3571	·4199	1810	−·3285	·4071
Length of Lactation Period on Yield of Butter Fat per week	14,416	−·3567	·4001	3179	−·3403	·4394	2820	−·3640	·3718	2233	−·3571	·4268	1810	−·3285	·4175
Percentage Butter Fat on Yield of Milk per week	14,416	−·0796	·1455	3179	−·0717	·1883	2820	−·0599	·1681	2233	−·0479	·2143	1810	·0007	·1346
Yield of Milk per week on Percentage Butter Fat	14,416	−·0796	·1279	3179	−·0717	·1754	2820	−·0599	·1091	2233	−·0479	·1475	1810	·0007	·0767

(2) *Yield of Milk per Week on Age.* The regression curve of milk yield per week on age for all cows (Diagram LXXVII) is very similar in character to the corresponding curve for the selected group. The mean yield rises from 17 gallons at 3 years of age, to 20 gallons at 10 years of age, after which the mean yield falls. At 18 years of age the mean yield is about 18·8 gallons.

(3) *Yield of Butter Fat per Week on Age.* The mean yield of butter fat per week ranges from about $6\frac{3}{4}$ pounds at 3 years of age to over $7\frac{1}{2}$ pounds at 9 years of age, when the mean value gradually falls and at 18 years of age the mean yield is $6\frac{3}{4}$ pounds. Thus, the mean yield of butter fat per week for all cows is, at all ages, greater than the corresponding mean yield of butter fat for the selected group (see Diagram LXXVIII).

(4) *Length of Lactation Period on Age.* The mean number of weeks in milk ranges from over 34 weeks at 3 years of age to 37 weeks at 18 years of age. The regression is linear and the rise, if not wholly due to, is probably accentuated by the elimination of younger cows with relatively shorter lactation periods and the retention of good cows with relatively long lactation periods (see Diagram LXXIX).

(*d*) *Heifers.*

A separate analysis was made of the mean yield of milk and the mean yield of butter fat per week for the 4776 Ayrshire heifers. The table below (Table XXXIX) shows the results. It will be seen that there is very little difference between the mean yield of milk per week for two year old heifers and for three year olds, but four year old heifers give a significantly higher mean yield

TABLE XXXIX.

Ayrshire Heifers—Total 4776.

Mean Yield of Milk per week (*Gallons*).

2 years old—15·931
3 years old—15·887
4 years old—16·903
All ages —15·909

Mean Yield of Butter Fat per week (*Pounds*).

2 years old—6·302
3 years old—6·286
4 years old—7·597
All ages —6·296

per week. The same is true with respect to the mean yield per week of butter fat. Two and three year old heifers are very much alike with respect to this character, but four year old heifers give a significantly higher yield.

DIAGRAM LXXVI.

1920 Records. Ayrshire Cows. Total, 14,416.
Regression of Percentage on Age.

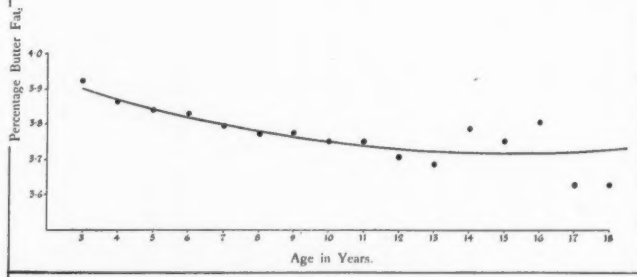


DIAGRAM LXXVII.

Regression of Milk Yield per Week on Age.

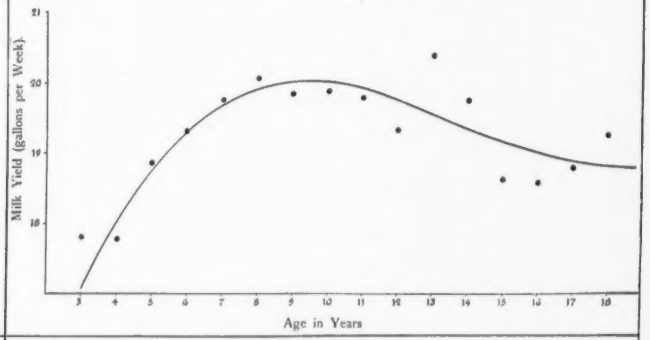


DIAGRAM LXXVIII.

1920 Records. Ayrshire Cows. Total, 14,416.
Regression of Yield of Butter Fat per Week on Age.

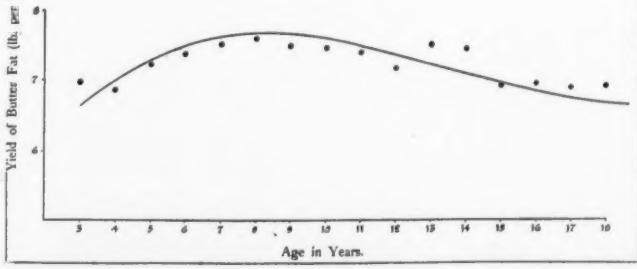
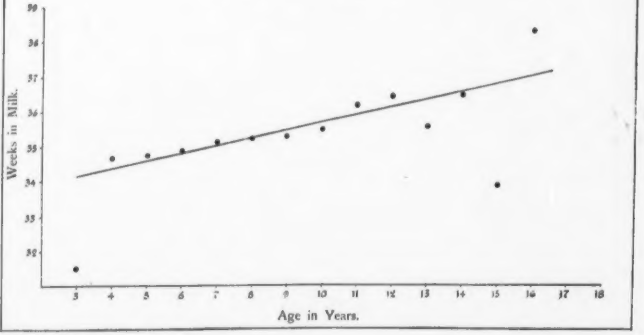


DIAGRAM LXXIX

Regression of Weeks in Milk on Age.



(7) *Comparison of Private with Association Records.*

(*a*) *Thirty-five Cows calving in April—Yield.*

The writer now proposes to examine in detail the methods employed by the Scottish Milk Records Association in determining the total yield of milk of each cow during its lactation period. In studying the variation in yield of a single cow during a lactation period, it is clear that a cow would give a record which would vary in a definite manner according to the month of calving. One would not expect, for instance, that a cow calving, say, in October would give, in successive fortnights, yields of the same amounts as a cow of the same age and breed calving, say, in April. There is usually an increase in milk yield when cows are put on to grass and the cows might be put on grass early in the lactation period, or in the middle, or towards the end of the lactation period. In order, therefore, to make a proper comparison between the estimated yield (by the Association) and the actual yield from private records for each fortnight during a lactation period, 35 cows were chosen out of a total of 107 cows whose daily yields were recorded because these 35 cows had all calved in the same month, namely, April. Variations during the lactation period would thus be similar in character. For example, they would all be put on grass at the same time.

It seems desirable at this stage to explain fully the method (the old method) of estimating the yield employed by the Milk Records Association during the period when the above private observations were made (1911–13). The method of estimating the yield employed by the Association from the year 1914 onwards is also given (the new method), as the object is to ascertain which of the two methods is the better when compared with the actual yields obtained from private sources.

In order to estimate the total yield for each fortnight during a lactation period under the old method (*pre*-1914) the Association's Recorder multiplied the yield on each day of test by the number of days which had elapsed since the previous test. This interval in the case of the 35 cows which calved in April was 14 days, and the product "daily yield × the number of days" represented the estimated total for 14 days preceding the day of test. In the case of the first test after calving, however, the yield was estimated differently. The daily yield was multiplied by the number of days between the *date of calving* and the date of the first test. The Association lactation period was considered complete on the date of the last Association test, irrespective of the fact that in most cases the cow would continue to give milk for several days after that date (in some cases over a fortnight), while in other cases the cow would become dry very soon after the last test. Of course in Association tests a mean date of some sort must be taken as the end of the lactation period.

A point to be considered is whether the yield of cows from the date of calving until the disappearance of colostrum in the milk should be included in the total milk yield during a lactation period. Those who advocate that it should not be included base their argument on the fact that colostrum is not marketable milk, and the interest of the members of the Scottish Milk Records Association centres round

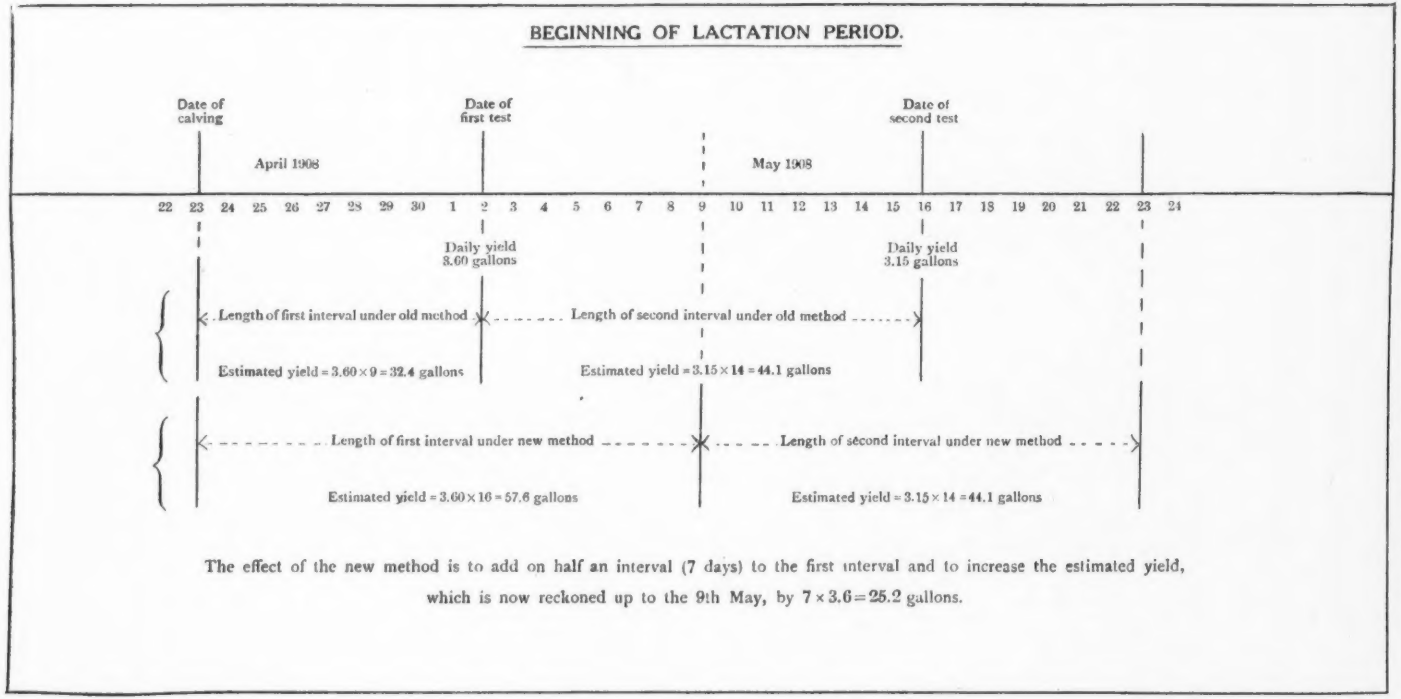
marketable milk. Those who advocate that the yield from the date of calving should be included point out that the colostrum is necessary for the calf and should therefore be included. In the cases examined, no private record was kept of milk yield from date of calving until four or five days after calving, when the colostrum had practically disappeared. Taking the view that marketable milk should be recorded, the writer looks upon the private records, which were very carefully kept, as being correct records of the actual marketable milk yield from each cow.

In order to estimate the total yield during the interval between the tests by the new method which has been employed since 1914, the estimate, "total yield"="daily yield" multiplied by "the number of days in the interval," is retained. The estimated yield is, however, looked upon as derived from a daily yield obtained at the middle of the interval between the two tests. In other words, this middle value is looked upon as the average value for the interval, and therefore, when multiplied by the number of days, gives a better estimate for the period, the half of which is in front of the day of test and the other half of which is after the day of test. The effect of this is to leave all the estimated yields unaltered except the estimated yields for the interval of the first test. Under the old method, the yield up to date of first test was estimated by multiplying the daily yield on day of test by the number of days between the date of calving and the date of first test. Under the new method, however, the second interval is reckoned from a date half the interval in days before the second test is made. The half period after the date of the first test is not overlooked. By the new method it is added on to the first interval. Thus the estimated yield for the first interval runs from the date of calving to half an interval beyond the date of the first test. For example, the interval of the first test would be *seven* days longer under the new method if the Association tests were carried out *fortnightly*. The end of the lactation period by this method is always half an interval beyond the last test. In general, therefore, the lactation period under the new method is always half an interval longer than it would be under the old method. The net effect is that the yield is increased, not by this prolongation at the end of the lactation period, but by the product "half an interval" multiplied by "yield on day of first test," as will be seen by the diagrams (Diagrams LXXX and LXXXI) which show, in the case of *fortnightly* tests, an actual example of the two methods.

Table LIX of Appendix shows for the 35 cows the dates of calving, dates of first and final tests, total yields and differences between Association and private yields.

The results (Table LIX, Appendix) show that both by the old method and the new method the Association estimate is in excess of the actual yields obtained from the private records. In the case of the old method, however, the excess is neutralised by the deficiencies in the estimates in various fortnightly intervals during the lactation period, while by the new method there are no such deficiencies—all the estimates are in excess. The actual total yield in gallons of the 35 cows from private daily records was 27148·6. The actual average yield for each cow (private records) was thus 775·67, while the corresponding figures for old

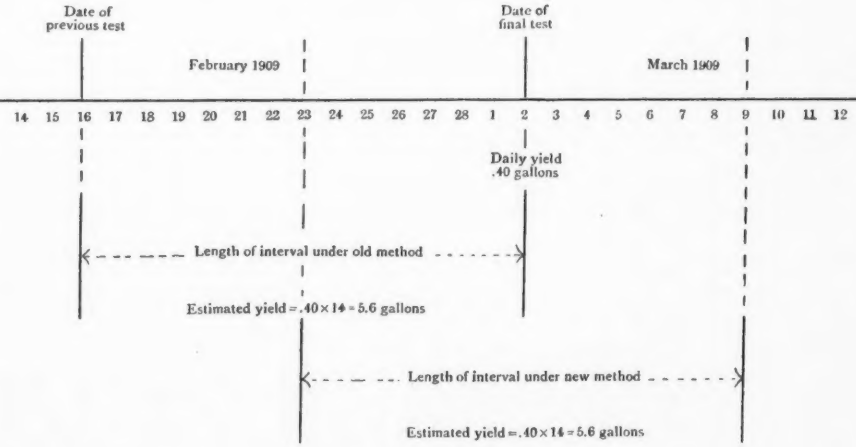
DIAGRAM LXXX.


BEGINNING OF LACTATION PERIOD.
Date of calving
Date of first test
Date of second test
April 1908
May 1908
22 23 24 25 26 27 28 29 30 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24
Daily yield 3.60 gallons
Daily yield 3.15 gallons
Length of first interval under old method
Length of second interval under old method
Estimated yield = 3.60 × 9 = 32.4 gallons
Estimated yield = 3.15 × 14 = 44.1 gallons
Length of first interval under new method
Length of second interval under new method
Estimated yield = 3.60 × 16 = 57.6 gallons
Estimated yield = 3.15 × 14 = 44.1 gallons

The effect of the new method is to add on half an interval (7 days) to the first interval and to increase the estimated yield, which is now reckoned up to the 9th May, by $7 \times 3.6 = 25.2$ gallons.

DIAGRAM LXXXI.

END OF LACTATION PERIOD.


Date of previous test
Date of final test
February 1909
March 1909
14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 1 2 3 4 5 6 7 8 9 10 11 12
Daily yield .40 gallons
Length of interval under old method
Estimated yield = .40 × 14 = 5.6 gallons
Length of interval under new method
Estimated yield = .40 × 14 = 5.6 gallons

This shows that no change is effected by the new method on the estimated yield derived from the daily yield obtained on the date of the final test.

and new Association methods are 776·58 and 801·54 respectively. The total Association yield for the old method was 27180·3 gallons, or an excess of 31·7 gallons for 35 cows which gives an average excess per cow of 0·906 gallons. The total Association yield by the new method is 28053·8 gallons, an excess of 905·2 gallons for 35 cows, or an average excess of 25·86 gallons per cow. *It is thus clear that if the old method is contrasted with the new method, the method now in vogue, the old method gives a better estimate of total yield for the lactation period for fortnightly tests.*

The average fortnightly yields for the 35 cows, both private and Association, were then determined. Table LX of Appendix shows the average fortnightly yields for the 35 cows from the second fortnight onwards, the first and last tests being omitted as requiring special study by themselves. By the old method, both the actual yields and the Association estimates show the average yields for the 35 cows per fortnight from a date 14 days before the second test to the date of the twenty-second test. By the new method, both in the private records and the Association estimate, the period runs *from seven days before the date of the second test* to seven days after the date of the twenty-second test. It will be seen from the table that the old method is an underestimate to the extent of 9·59 gallons for the period. Since the old method gives a good estimate for the whole lactation period, it is clear that this is due to the overestimate for the period of the first date balancing the underestimate for the remainder of the lactation period. It will be seen that the estimate by the new method is, on the other hand, an overestimate to the extent of 10·07 gallons. This average overestimate, added to the overestimate for the first test, accounts for the average excess by the new method of the estimated yield of 25·86 gallons for the actual average yields.

These results suggest another method of approaching the problem, namely, to estimate the milk yield from a date seven days before the *second* fortnightly test to seven days after the last test, and to fit regression curves to the data. The estimate from the date of calving to the date seven days before the second fortnightly test is left out in order to eliminate the inaccuracies associated with the first test. What is marketable milk? The change from colostrum to marketable milk takes place in about five days after calving, but individual cows vary in the period in which they cease to give a milk with colostrum properties. Albumen and globulin are present on the first day after calving to the extent of about 16 per cent. in colostrum, while there is more fat and less sugar than in ordinary milk. It will therefore be useful to consider what results would be obtained if we were to measure the milk-yielding capacity of a cow from a period *where the data are reliable with respect to days.* If this process is applied both to estimated and actual yields the results would be strictly comparable. The results are seen in the following tables and diagrams, which show the actual and estimated fortnightly yields. Quartic curves were fitted to the data. The results are those one would expect from an indefinitely large population of cows, of which the 35 cows are a random sample (see Diagrams LXXXII and LXXXIII). The results are given in Table XL which should be studied along with the accompanying key.

DIAGRAM LXXXII.

Variation in Fortnightly Yield during Lactation Period (Private Records).
The Curve fitted to the Actual Yields is a Quartic Curve.

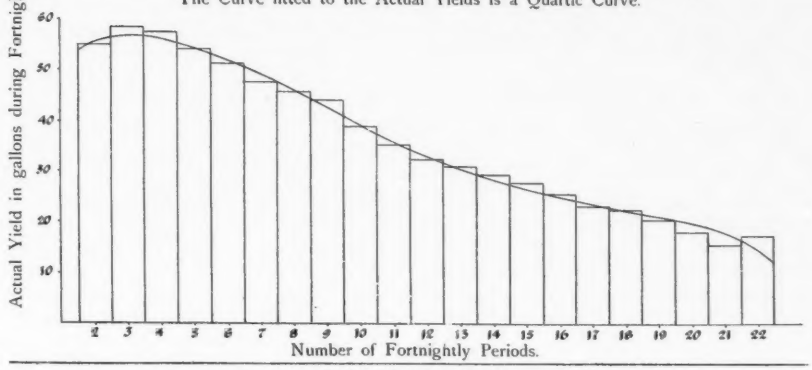

Actual Yield in gallons during Fortnight.
60
50
40
30
20
10
2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22
Number of Fortnightly Periods.

DIAGRAM LXXXIII.

Variation in Fortnightly Yield during Lactation Period (Association Estimate).
(Quartic Curve fitted to observations).

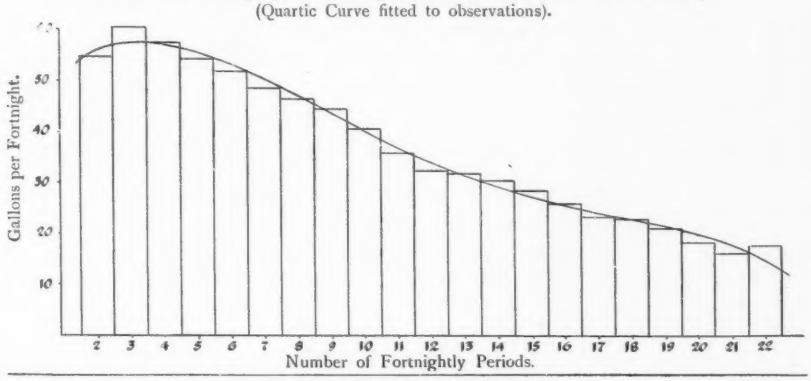


DIAGRAM LXXXIV.

Showing that the Estimated Yield is always in excess of the Yield found
from Daily Private Records.

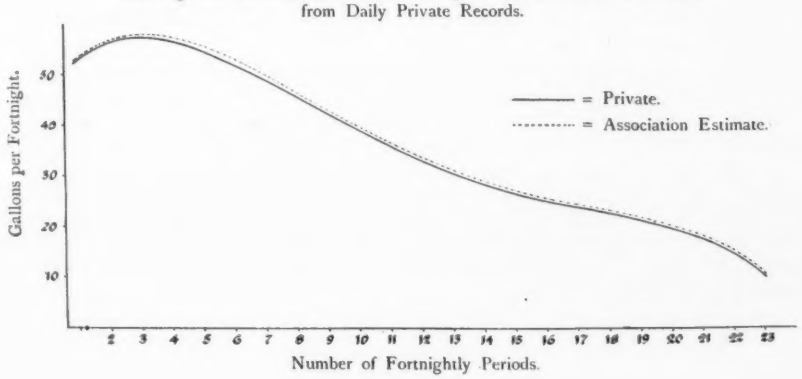

Gallons per Fortnight.
50
40
30
20
10
——— = Private.
····· = Association Estimate.
2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23
Number of Fortnightly Periods.

TABLE XL.

Showing the Average Fortnightly Yields per Cow (in Gallons) by Association's Estimates and the Actual Fortnightly Yields from the Private Daily Records from Second Fortnightly Test onwards.

No. of Fortnightly Tests	(1) Observed Results from Daily Private Records	(2) Quartic Curve fitted to figures in column (1)	(3) Sum of Daily Yields for 14 days from Quartic Curve of column (2)	(4) Calculated Fortnightly Yields from Association's Fortnightly Single Test at Middle of Fortnight	(5) Quartic Curve fitted to figures in column (4)	(6) Differences between column (5) and column (2)	(7) Differences between column (4) and column (1)
2	55·0	55·63	55·6	54·9	55·94	·31	−·1
3	58·5	56·71	56·7	60·3	57·17	·46	1·8
4	57·5	56·30	56·3	57·4	56·86	·56	−·1
5	54·3	54·74	54·7	54·4	55·37	·63	·1
6	51·1	52·34	52·3	51·7	53·01	·67	·6
7	47·7	49·38	49·4	48·4	50·06	·68	·7
8	45·7	46·09	46·1	46·4	46·76	·67	·7
9	44·0	42·67	42·7	44·4	43·32	·65	·4
10	39·2	39·30	39·3	40·3	39·91	·61	1·1
11	35·3	36·11	36·1	35·8	36·67	·56	·5
12	32·2	33·19	33·2	32·4	33·69	·50	·2
13	31·0	30·60	30·6	31·6	31·06	·46	·6
14	29·6	28·37	28·4	30·2	28·78	·41	·6
15	27·9	26·49	26·5	28·2	26·86	·37	·3
16	25·6	24·92	24·9	25·9	25·25	·33	·3
17	23·3	23·56	23·6	23·4	23·88	·32	·1
18	22·5	22·31	22·3	22·8	22·62	·31	·3
19	20·4	21·02	21·0	21·1	21·35	·33	·7
20	18·0	19·50	19·5	18·4	19·85	·35	·4
21	15·5	17·51	17·5	16·1	17·92	·41	·6
22	17·2	14·82	14·8	17·5	15·30	·48	·3
Totals	751·5	751·56	751·5	761·6	761·63	10·07	10·1

KEY TO TABLE XL.

Column (1) gives S/N where S=sum of daily records for 35 cows for 14 days from a date seven days before to a date seven days after the Association's test in each case; and N=35 cows. Thus column (1) shows the actual average fortnightly yield per cow from the second fortnightly test for 35 cows calving in April.

Column (2). These figures show the theoretical values for column (1). They are the figures of the best fitting curve to the *observed* values in column (1). The figures are comparable with the figures in column (5), which show the theoretical fortnightly yields per cow from the Association's fortnightly tests, the observed values of which are given in column (4).

Column (3). The figures in column (3) are the figures obtained by summation of the theoretical average daily yields per cow for each fortnight. The equation to the quartic curve (private data) was integrated between the limits of 1·5 to 2·5; 2·5 to 3·5.....21·5 to 22·5 fortnightly periods. (See Diagram 82). It will be seen that the figures are practically identical with figures in column (2). They thus show that the figures in column (2) closely describe the average fortnightly yield per cow.

Column (4). The figures in column (4) give the results per cow for the same 35 cows, obtained by the Association's single test by multiplying the yield found on day of test by 14, reckoning the fortnightly period from a date seven days before to a date seven days after the day of test.

Column (5) shows the theoretical values from the best fitting curve to the observed association values in column (4). These figures are to be compared with the Private Record figures in column (2).

Column (6) shows the differences between column (5) and column (2) and reveals the fact that, even leaving out the results of the first and last Association tests which accentuate the differences, the Association's estimate is always in excess of the actual results (see also Diagrams LXXXII, LXXXIII and LXXXIV).

Column (7). The differences between column (1) and column (4) show a similar excess on the part of the Association's actual estimates over the *Actual* fortnightly yields from private daily records. In neither case, however, is the excess great for any fortnight so that the Association's estimates can be accepted in practice from the second fortnight onwards. As is shown, however, on page 200 and Diagrams LXXX and LXXXI, the Association's estimate for the *first* fortnightly test is responsible for the Association's additional exaggerated yield when compared with the actual sums of daily yields during the first fortnight.

The results clearly show that the excess of the Association's estimate of fortnightly yield is a real excess over the actual yield for every fortnightly period (see columns (6) and (7)), the total excess being 10·07 gallons. This is an error equal to 1·38 per cent. of the total yield compared with 3·33 per cent., the percentage excess estimate by the Association's new method when including its treatment of the first fortnightly test. In other words, the excess estimate brought about by using the Association's new method for the first fortnight amounts to 1·95 per cent., or 58·56 per cent. of the total excess. I cannot see that, apart from actually taking daily records, the Association can do better than take a test in the middle of the fortnightly period and multiply by 14 to give an estimate of the total yield for the fortnight even although it does involve a total average over-estimate of about 10 gallons from the second fortnightly period to the end of the lactation. If the milk recorder would make his first test say 12 days after calving, he would be allowing an average of five days for the colostrum period and the lactation period would begin five days after calving. I imagine this is not possible in practice and thus, if the most accurate estimated yields are to be used as the basis of the milking capacity of a cow, it is clear that the estimate should be based on the period *commencing seven days before the second fortnightly test.*

If the total yields are considered over the whole lactation period (see Appendix, Table LIX) the Association tests show for an individual cow a maximum defect by the old method of 17·7 gallons and a maximum excess of 26·9 gallons over the same period. If total yields from the private records are compared with the yields according to the Association's new method there is *always* an excess in the Association's figure over the private figure. For an individual cow the minimum excess is 1·6 gallons and the maximum is 52·3 gallons. The average difference among single cows between the yield from the private records and the Association's old method is 0·91 gallon in excess, while the corresponding figure when comparing the private records with the Association's figure by the new method is 25·86 gallons in excess.

Attention was now directed to the period of the first test. Table LXI of Appendix shows the actual yields from the private records up to the date of the first Association test (column (1)), and up to a period beyond the date of the first Association test corresponding to half the interval in days between the tests (column (2)). This latter method enables one to look upon the Association "yield on day of test" as being the value on the seventh day of the interval of 14 days and is therefore approximately the average for the 14 days.

If we concentrate our attention on the first fortnightly test during the lactation period for the 35 cows, we find that the actual average yield from the private records is 34·06 gallons by the old method, while the corresponding average from the Association estimates is 45·99 gallons, an average excess of 11·93 gallons. By the new method, the private records give an average of 60·21 gallons up to seven days beyond the first fortnightly test, while the Association figure is 70·99 gallons, an excess of 10·78 gallons for the corresponding period. The differences 11·93 and 10·78 gallons, respectively, are clearly due to the fact that the private tests began on the day the cow's milk was marketable, while by the Association tests the cow's yield is reckoned from the date of calving and is supposed to give every day, from date of calving to date of test, the yield found on day of test.

TABLE XLI.

Showing where the Underestimates and Overestimates occur during the Lactation Period.

A.

Estimate by Old Method.

			Gallons
Estimated Yield for Lactation Period			776·6
(1) Overestimate due to First Fortnightly Period		11·9	
(2) Underestimate, excluding First and Last Fortnightly Periods	7·2		
(3) Underestimate for Last Fortnightly Period	3·8		
	—	11	
(4) Nett overestimate by Old Method			0·9
Actual Yield			775·7

B.

Estimate by New Method.

Estimated Yield for Lactation Period		801·5
(1) Overestimate due to First Fortnightly Period	10·8	
(2) Overestimate, excluding First and Last Fortnightly Periods	10·1	
(3) Overestimate for Last Fortnightly Period	4·9	
(4) Total overestimate	—	25·8
Actual Yield		775·7

Table XLI shows where the underestimates and overestimates occur. Under the old method the nett overestimate (·9 gallon) is seen to be the difference between the average overestimate made on the first fortnightly period and the average underestimates for the remaining fortnightly periods. Under the new

method the total average overestimate of 25·8 gallons is seen to be due (1) to an average overestimate of 10·8 gallons made on the first fortnightly period, (2) to an average overestimate of 4·9 gallons made on the last fortnightly period, and (3) to an average overestimate of 10·1 gallons made during the intervening fortnights between the first and last periods.

(*b*) *Yield from Cows irrespective of Date of Calving.*

Hitherto the writer has considered cows calving in the same month. The yields as obtained from private and Association records will now be considered for cows calving in any month of the year. For this purpose the data of 107 cows were available. Table LXII of Appendix shows the total yields during a lactation period—private and Association—for these cows. The Association figures are as given in the records, the old method being the method in vogue when these tests were made.

The Association yield for a single lactation period shows a greatest defect of 29·9 gallons when compared with the private figure, and a maximum excess for a single lactation period of 57·1 gallons over the private figure. The actual mean yield from the private records was 792·06 gallons, while the mean yield from the Association's record was 793·59, or an excess of 1·53 gallons, which is the comparable figure to 0·906 gallon in the examination of the 35 cows. It has been determined that if seven days were added on to the first interval of each lactation period, being half the interval between each Association test, then the mean yield would be increased by 26 gallons, while the individual estimates for each of the 107 cows would be similarly increased.

A series of daily tests was carried out between 1914 and 1917 by the same observer who supplied the data from the 107 cows. Table LXV of Appendix gives the results of a statistical analysis of these daily tests.

It is seen that the actual average yield is 708·8 gallons, while the estimated average yield is 734·7 gallons, an excess by the Association's new method of 25·9 gallons. The estimated average yield by the old method is 707·4 gallons, an average deficiency of 1·4 gallons when compared with the actual average yield.

The last records examined were from the herd under the control of the West of Scotland College of Agriculture. These records were very kindly placed at my disposal for statistical analysis by Mr A. C. M'Candlish, M.S.A., of the Milk Production Department of the College. Mr M'Candlish and Mr M'Vicar had made a preliminary examination of the records and compared the actual daily yields of 24 cows with the product "yields on each tenth day × 10." The results were published in the *Scottish Journal of Agriculture**, and show that the sum of the estimated yields based on tests made every ten days is a good approximation to the actual total yield of each cow. These authors in the figures of their Table I show the estimated yields for intervals of 10, 20 and 30 days, *but they do not include the yield for the interval in days, between date of calving and date of first*

* *The Scottish Journal of Agriculture*, Vol. VIII. No. 2, pp. 201—205.

*record of each cow required by the old and new methods of the Scottish Milk Records Association**. Since Mr M'Candlish also kindly supplied the lengths of these intervals in days for all the 24 cows, the author has been able to calculate the estimated yields by the old and new methods, and these are given in Table LXIII of Appendix, together with the actual yields in each case. For comparative purposes the same data were used, selecting the recorded yields every fourteen days, the

TABLE XLII.

Summary of Results. Actual Yields and Estimated Yields.

Number of Cows	Actual Mean Yield. Gallons	Association's Estimates in Gallons			
		Old Method	Difference	New Method	Difference
35*	775·7	776·6	+0·9	801·5	25·8
107*	792·1	793·6	+1·5	819·6	27·5
18*	708·8	707·4	−1·4	734·7	25·9
24†	642·8	645·3	+2·5	663·0	20·2
184	761·36	762·59	+1·23	787·43	26·07
	Average values				

1 Number of Cows	2 Actual Mean Yield. Gallons	Association's Estimates in Gallons			
		3 Old Method	4 Difference columns 2 & 3	5 New Method	6 Difference columns 2 & 5
35*	775·7	776·6	+0·9	801·5	25·8
107*	792·1	793·6	+1·5	819·6	27·5
18*	708·8	707·4	−1·4	734·7	25·9
24*	642·8	637·3	−5·5	661·9	19·1
184	761·36	761·55	+·19	787·28	25·92
	Average values				

* Fourteen days' interval. † Ten days' interval.

corresponding total estimated yields for each lactation period being calculated by the Association's old and new methods respectively. The results are given in Table LXIV of Appendix. It will be seen that, with intervals of 10 days between the tests, the old method overestimates the actual yield on an average by 2·5 gallons, while the new method's overestimate is 20·2 gallons. With fortnightly intervals between the tests, the corresponding figures are averages of 5·5 gallons

* See page 205 of above.

less and 19·1 gallons more respectively. The greatest variations in the predictions by the old method with 10 days' interval are 8 gallons less and 43 gallons more than the actual yields, while the greatest excess prediction by the new method is 55 gallons. The corresponding figures for the fortnightly test are, by the old method, 10 gallons less and 63 gallons more, respectively, for a lactation period. There are no underestimates by the new method, the greatest overestimate being 98 gallons and the least overestimate 15 gallons.

We are now in a position to show the results of calculations of total yields according to the Association's old and new methods of four herds for comparison with the actual total yields obtained by summing the observed daily yields carefully recorded in each case.

The results from 184 cows are summarised in Table XLII. The old method shows an overestimate on the 10 days' test of 1·23 gallons. By the new method the overestimate is 26·07 gallons. The corresponding figures for the fortnightly test show an overestimate of 0·19 gallon by the old method, and an overestimate of 25·92 gallons by the new method.

It is clear from the foregoing results that the present method of estimating milk yield practised by the Scottish Milk Records Association (called herein, the new method) when applied to fortnightly tests consistently overestimates the actual yield. The old method, practised by the Association prior to 1914, gives better results. Our investigation shows that a revision of the method of estimating milk yields is necessary.

IV. Summary of Results.

(1) The results of this investigation are the results of an analysis (1) of the whole of the records of the Scottish Milk Records Association for the years 1911, 1912 and 1920, and (2) of certain sections of the records for the years 1908, 1909 and 1923. The results for the years 1911, 1912 and 1920 are from records in which the dates of birth of calves are given both at the opening of the lactation period and at its close. This does not mean, of course, that all the cows were in milk right on to the beginning of the next lactation period. As will be seen from the tables, the lactation periods of many cows were quite short, others were of average duration, while a certain number of cows gave milk almost to the beginning of the next lactation period. The general results of analysis of the data of 1911 and 1912, together with an inquiry into the extent of selection between 1911 and 1923, are discussed in pages 108 to 165, while the corresponding results for the 1920 data and comparisons with 1908, 1909 and other data are discussed from page 165 onwards.

(2) The mean yield for a lactation period of 36·78 weeks in 1911 was 634·82 gallons. The mean yield for 1912 for a lactation period of 37·41 weeks was 654·20 gallons, while the yield for 1912 with an average lactation period of 36·78 weeks (the mean figure for 1911) was found to be 643·26 gallons. There is thus a real average increase in yield of 8·44 gallons in 1912 compared with the average value for

1911. It is shown that the difference is significant. Since the average percentage of butter fat is practically the same for both years, there is a significant increase in 1912 in the average total yield of butter fat during a lactation period, 244 pounds, to the extent of about 8 pounds when compared with the corresponding figure, 236 pounds, in 1911. This increase corresponds to approximately 10 pounds of butter.

(3) The distributions of the five characters studied, namely: Yield of milk during a lactation period (Q); average percentage of butter fat during a lactation period (P); Age of cow (A); Length of lactation period in weeks (W); and total yield of butter fat during a lactation period (B); have been studied and Pearsonian curves fitted to the frequency data in each case. All the five characters show considerable variation round their average values, the coefficients of variation ranging from 8·7 to 43·4.

For the distribution of quantity for each year of age it is shown that Type IV curves adequately describe the observations and that the fits are good considered as a whole (see Table VI and Diagrams III and IV). One of the curves is a rather poor fit. Type IV curves have also been fitted to percentage of butter fat, weeks in milk and yield of butter fat. If the total butter fat yield is considered for cows all of one age (3·5 years was selected) and milked for a constant number of weeks (41 weeks) it is found that a normal curve fits the distribution very well (see Table VII).

(4) It is shown that the relationships between quantity and average percentage of butter fat, and between percentage of butter fat and average quantity are practically linear. In other words, a low yield has a high average percentage of butter fat, and the average percentage slowly and uniformly falls with increase in yield. Average yield also continuously and uniformly falls with increase in percentage of butter fat. Thus, cows with high yields for a lactation period have a lower average percentage of butter fat than cows with low yields.

(5) It is seen that cows giving large yields during a lactation period are less variable among themselves with regard to butter fat than cows giving low yields. Further, it is shown that cows giving high average percentages of butter fat during a lactation period are less variable among themselves with respect to average quantity than cows giving low percentages of butter fat. Low percentage of butter fat indicates a comparatively great variability in yield, while high percentage of butter fat indicates a comparatively small variability in yield. Thus, if cows were selected directly for high average percentages of butter fat during a lactation period, there would be a direct tendency to uniformity of yield. Further, if cows were selected for high yield alone, this would bring about a direct tendency to uniformity in the average percentage of butter fat among the high yielders.

(6) The regression of average quantity on age is non-linear. Regression parabolae have been fitted to the data for both years. The fit is good up to 12 years of age, but there are greater deviations from the curve beyond 12 years which are largely accounted for by the smallness of the numbers. It will be seen that there is a constant increase in the total yield for a lactation period up to

about 13 years of age, after which the yield decreases with age. The regression parabolae as shown on Diagrams XIV and XV and the relevant tables can be used to furnish valuable information to the dairy farmer. This question might be put: From the known results regarding quantity and age, what is the total yield of milk I might expect to get during a lactation period from a herd of, say, 10 cows, the ages of which are given? The answer is supplied in Table VIII, and the milk producer can compare this result with the actual results from his 10 cows. He would then be able to see whether his herd is doing better or worse than the general population of dairy cows.

(7) The relationship between age and percentage of butter fat during a lactation period is negative as measured by the correlation coefficient. The regression of average percentage on age is, however, non-linear. There is a gradual fall in the average percentage of butter fat with increasing age until about 11 years, after which the average fat-percentage rises with age. In other words, the *age-percentage* curve is concave while the *age-yield* curve is convex. The minimum average fat-percentage and the maximum average yield occur about 11 years and 13 years respectively. The dairy farmer can find the average percentage of butter fat associated with any age of a herd by referring to Diagrams XVI and XVII, just as he can find the average yield associated with any age from Diagrams XIV and XV.

(8) The highest correlation between any two characters is that between quantity and length of lactation period—the mean value being about 0·6. The regression of average quantity on weeks is practically linear, any departure from linearity being at very short and very long lactation periods.

The regression of average weeks on quantity is non-linear in character. The 1912 data, however, show that for quantities between 450 and 1000 gallons the regression is linear.

(9) The regression of average percentage of butter fat on weeks in milk is non-linear in character, but the departure from linearity is not very great. The results show that dairy cows with long lactation periods give, on an average, a higher average percentage of butter fat during a lactation period, when compared with the results from dairy cows with a short lactation period. It is also shown that a group of cows with a high percentage of butter fat is associated with a long average lactation period when compared with a group of cows giving a low percentage of butter fat.

(10) It is shown that the regression of average total yield of butter fat for a lactation period on age is parabolic, as was to be expected from the yield of milk and age results. The age at which the maximum amount of total butter fat for a lactation period is produced is 10 to 11 years (1911). Young cows have a total butter fat yield of about 200 pounds for a lactation period. The total yield of butter fat increases continuously with age until at about $10\frac{1}{2}$ years approximately 260 pounds is produced (1911 data) and over 270 pounds (1912 data). After $10\frac{1}{2}$ years there is a steady fall with age. At 18 years of age, the yield of butter fat is less than 240 pounds (1912 data).

(11) It is of first-class importance to the dairyman to know the total quantity of butter fat likely to be produced during a lactation period for a cow of a definite age, for example, say, 7 years, with a definite length of lactation period, for example, say, 41 weeks. Table XII of the memoir shows that the expected quantity is 289·9 pounds. The observed quantity for cows of 7 years old and milked for 41 weeks was 290 pounds. Dairy farmers desiring to find the maximum amount of butter fat to be expected from their cows can do so by using this table. They will note that with younger cows, the yield of butter fat increases with age and with length of lactation period and, unless a cow is an exceptional butter fat producer, they will know from the table when to cease using the cow for dairy purposes. The attention of the reader is directed to the model prepared from the data in Table XII. It will be noted that the agreement between the theoretical surface and the observed values is of a satisfactory character.

(12) It is shown that about 50 per cent. of the dairy cows for 1912 are classed as "good" by the Milk Records Association; 45·5 per cent. as "intermediate" and over 4 per cent. as "bad." In other words, the Association places 50 per cent. of all cows into a class yielding 250 pounds and over of butter fat for a lactation period; over 45 per cent. into a class yielding between 166 and 250 pounds; and 4 per cent. into a class giving a total butter fat yield of less than 166 pounds. Over 67 per cent. of all heifers fall into the Association's "good" class, that is, those heifers giving a total yield of butter fat of 200 pounds and over; over 29 per cent. fall into an "intermediate" class where the total yield of butter fat is between 133 and 200 pounds; and over 3 per cent. fall into the "bad" class, that is, heifers giving a total of less than 133 pounds of butter fat for a lactation period. The Association have not strictly adhered to these divisions in recording the data, as will be seen from Diagrams XXIX and XXX and Table XIV. The figures in Table XV show the mean values for the various characters for the three classes of cows and the three classes of heifers.

(13) A new method of classification is suggested by the writer in section 5 (*b*), the standard deviation being taken as the unit for classification purposes. It is suggested that cows giving a total yield of butter fat between the average and the yield represented by (average + once the standard deviation) be regarded as one class, those above this value and up to a quantity represented by the (average + twice the standard deviation) be regarded as another class. Other methods of classification are suggested.

(14) A study is made of the nature of the changes in the correlation coefficients of the various pairs of characters when the good, intermediate and bad classes are considered separately instead of the whole population of cows (see Table XX and the text). An analysis was also made of the 1911 and 1912 combined data, in which the cows and heifers were divided into four age groups (see Tables XXI—XXII). It is seen that for a constant number of weeks the correlation between total yield of butter fat and quantity is very high for all groups, the correlation coefficient ranging from 0·85 to 0·89.

(15) An investigation was made of the combined 1911 and 1912 data after subdivision into three age groups, namely: Group I, 2—3 years inclusive; Group II, 4—8 years inclusive; Group III, 9—18 years inclusive. Group III is found to be most variable and Group I is the least variable. Group III has the greatest mean yield of milk and the greatest mean total yield of butter fat. Selection may here be a factor, as Group III is the group of oldest cows where many have been kept because they are good cows (see Table XXI), thus intensifying the correlation between yield of milk and age.

(16) It is known that dairy farmers frequently reject young cows which prove to be bad milkers. Since the milk characters have all been shown to be more or less correlated with age of cow, it is clear that the population of dairy cows being studied is a selected population with respect to age and other characters. In other words, the dairy cows are selected in the sense that a certain number of low yielders are excluded. The population is, therefore, not a random population and it might be contended that the whole, or practically the whole, of the relationship between age and quantity or between age and yield of butter fat is due to the fact that certain older cows are retained for milking purposes and are thus positively selected because of their higher yields, while certain other older cows have been rejected and are thus negatively selected because of poor yields. It appeared necessary, therefore, to study the data in order, if possible, to determine whether a significant correlation also existed between age and the above-named characters in the *general unselected* population of Scottish dairy cows. It is evidently not possible to make the determination directly on a random sample of the general population of cows, on account of the fact that large numbers fail to get into the records. Professor Karl Pearson, however, in his memoir on "The General Theory of the Influence of Selection on Correlation and Variation*," has shown how to find the correlations in a random sample of an unselected population from selected data. The method by which the writer has approached this problem is shown in section (6) of this memoir. Owing to the nature of the data, it did not appear possible to find the exact correlations. The results in Table XXV, however, show the limits within which the correlations lie. The results of section (6) show that age and quantity, and age and total yield of butter fat are significantly correlated in a *general unselected* population of Scottish dairy cows. In other words, quite independently of any cutting off of poor cows by dairymen, there is a definite trend towards increased quantity and increased yield of butter fat as the cow gets older. When, however, the cow reaches the age of from 10 to 11 years, the trend is in the direction of less quantity and less butter fat yield with increase in age of cow.

(17) It is shown that the average yields per single milking at various ages of cows taken at random are not comparable with the average yields per milking for cows of various ages, the averages being taken for entire lactation periods (see Diagram XLI A). It is further shown that the average yield per week is practically

* *Biometrika*, Vol. VIII. pp. 437 *et seq.*

constant after cows have been in milk for 26 weeks. An attempt has been made to measure the intensity of selection between 1911 and 1920. It is shown that when the 1920 records are made comparable with the 1911 records there is an increase to the extent of over 4 per cent. in the average yield of milk between 1911 and 1920. The selection of good cows was also considered, good cows being defined as animals giving over the equivalent of 2500 gallons containing 1 per cent. of butter fat for the lactation period. It is shown that the average increase per week in yield among good cows amounts to ·63 of a gallon. It is further shown that this is not a random increase but that the good cows in 1923 give higher yields on an average than good cows in 1912.

(18) A study of the 1920 records shows that considerable changes have taken place in the interval between 1912 and 1920 in yield and certain other characters of the milk supply as observed by the Scottish Milk Records Association, but it should be borne in mind when studying the results and particularly the figures in Table XXVIII that the 1920 figures are from data calculated by the new method while the 1911 and 1912 figures are from data calculated by the old method. In the previous paragraph (17) the extent of the *real* change has been shown. The *apparent* changes are as follows: The yield of milk for a lactation period rose from 635 gallons in 1911 to 697 gallons in 1920. The total yield of butter fat during a lactation period rose from 236 pounds in 1911 to 266 pounds in 1920. The yield of butter fat per week rose from 6½ pounds in 1911 to 7 pounds in 1920. There is little change in the average length of lactation period as between 1911 and 1920. There is a gradual rise, however, in the average percentage of butter fat during a lactation period, when the records of 1908 to 1920 are considered (see Table XXVIII).

(19) The data kindly supplied by Dr G. W. Monier-Williams with respect to butter fat have been analysed and the results compared with the results of this investigation. It is shown that the best-fitting curves to the Monier-Williams' distribution are Type IV curves. Only in one case was a Type I curve indicated, while in another case a normal curve proved to be a good fit. It should be pointed out that his results are from a large herd of cows, 65 in number. Table XXX shows the mean percentages of butter fat for morning and evening milk, and when cows are grass fed or are on winter rations. The percentage of butter fat varied from 3·17 in the case of grass fed cows of the Moundsmere herd, to 4·24 in the case of the same herd when on winter rations. The other results in Table XXX show that butter fat varied from 3·56 per cent. (No. 10 Hull) to 3·98 per cent. in the case of No. 8 Lanark.

(20) The distribution of weekly yield for each year of age was studied in order to ascertain the nature of the distributions. The results are given in Table XXXI and show that Type I, Type II and Type IV curves are indicated. The fits in all cases are excellent, the lowest value of P being ·37 in the case of four year old cows. The mean yield per week varied from 16·88 gallons in the case of three year old cows to a maximum of 19·95 gallons in the case of eight year old cows. The yield very gradually fell to 19·13 in the case of cows twelve years old and over.

(21) The variability of yield per week increases with age from 3·1 gallons at 3 years of age to 3·6 gallons at 12 years of age (see Diagram LXX). Older cows are therefore more variable in the milk yield than young cows.

(22) An analysis of the 1908—1909 data with respect to yield and age and variability of yield and age has been carried out and compared with the results of the analysis by Professor Raymond Pearl. The differences between the standard deviations of yield and the mean array standard deviations of yield on age are discussed. It is pointed out that the differences are not wholly due to the changes in the mean weekly yield with age. Another factor operates to affect the change, namely, the variability of the yield array standard deviations on age. Taking one particular district, it is shown that the variances of the array standard deviations of yield on age for Fenwick are higher than the corresponding figure for all Ayrshires. Thus the cows of the Fenwick district show greater variability in weekly yield as the cows get older than Ayrshires in other districts do. It is shown that there is a distinct fall in the correlations between age and yield in 1920 as compared with 1908 and 1909. Approximately the correlations in 1920 are half the correlations in 1908 and 1909.

(23) The values of the correlation ratios and coefficients for all characters are shown in Table XXXV. The highest correlation is found between yield of milk per week and age. The markedly non-linear regressions are yield of butter fat per week on age, and yield of milk per week on age. Diagrams LXXII to LXXV show the regression curves for the pairs of characters studied for 1920.

(24) An analysis of the data was carried out in order to ascertain whether or not there is any significant difference in milk yield and other characters studied for cows calving at different months of the year. A reference to Table XXXVI shows that cows calving during the months of August, September, October and November (Group C) have a significantly longer lactation period than cows calving during other months. The percentage of butter fat, yield of butter fat per week in pounds and yield of milk per week in gallons, are approximately the same for all months of the year. Owing to the fact that cows calving in August, September, October and November (Group C) have a longer average lactation period than for other months, the total yield for an entire lactation period is higher for cows calving in these months when compared with the remaining eight months.

(25) A study was made of the records of 14,416 cows whose records are given for 1920 in order to compare the results with the 4912 cows already studied. As might be expected, the mean length of lactation period was shorter in the case of the 14,416 cows when compared with the 4912 cows, owing to the fact that some of the lactation periods of the former are incomplete.

(26) The values of the correlation coefficients and ratios for the various characters on age of the 14,416 cows are given on Table XXXVIII for cows aged 4, 5, 6 and 7 years respectively and for cows of all ages. The regression curves are shown in Diagrams LXXVI to LXXIX.

(27) A separate analysis was made of the mean yield of milk and the mean yield of butter fat per week for 4776 Ayrshire heifers. The results are given in Table XXXIX.

(28) The writer was fortunate to receive from a prominent member of the Committee of the Scottish Milk Records Association a large number of private records of milk yields of his herd, where the milk yield was accurately determined for each cow every day during the lactation period. The actual yields from private records were thus available for comparison with the estimated yields from the *fortnightly* tests made by the recorder of the Association. In order that a proper comparison could be instituted, the records of 35 cows which had calved during the month of April were first examined, and the results compared with the estimated results by the old method used by the Association up to 1914, and by the new method used by the Association since 1914. The actual total yield in gallons of the 35 cows was found from the private daily records to be 27,148 gallons, while the total estimated yield by the Association's first method was found to be 27,180, or an excess of 32 gallons for the 35 cows, equal to an average excess of 0·9 gallon per cow. The estimated yield by the Association's second method was found to be 28,053 gallons, or an excess of 905 gallons for 35 cows, equal to an average excess of nearly 26 *gallons per cow.* It is thus clear that the old method gives better estimated results than the new method. The reasons for this are fully explained in section III (7). It is shown that the new method gives worse results than the old method because seven days are added to the total number of days in milk, while at the same time the estimate is made from the date of calving. The conception that the test can be regarded as being taken in the middle of a fortnightly period instead of the end could be adopted without adding to the number of days of the lactation period. The article of M'Candlish and M'Vicar is discussed and it is shown that the reason why their figures appear to give better results by the new method is because they have not really estimated the yields by the Association's method at all. They have omitted a number of days which should be included if the Association's method of estimation is properly carried out. Until data of private tests from a number of herds are available for comparison with the Association's fortnightly tests, it is clear that the Milk Records Association should go back to the old method of estimating total yields by means of fortnightly tests as giving more accurate results than the new method.

APPENDIX OF TABLES

TABLE I. 1911 *Data. Quantity and Percentage* (Diagrams VIII and X)

Percentage of Fat

Quantity of milk in gallons Above	Up to	2·75 Above	3·00 Up to	3·25 Above	3·00 Up to	3·50 Above	3·25 Up to	3·75 Above	3·50 Up to	4·00 Above	3·75 Up to	4·25 Above	4·00 Up to	4·50 Above	4·25 Up to	4·75 Above	4·50 Up to	4·75 and over	Totals
	200	—	—	1	—	—	1	—	—	1	—	—	—	—	—	—	—	—	2
200	250	—	—	4	1	3	3	1	—	—	—	—	—	—	—	—	—	—	12
250	300	—	1	—	5	4	5	3	1	—	—	—	—	—	—	—	—	—	19
300	350	—	3	10	16	14	9	4	1	1	—	—	—	—	—	—	—	—	58
350	400	2	2	12	41	43	25	11	2	2	—	—	—	—	—	—	—	—	140
400	450	—	11	36	81	71	50	21	5	2	—	—	—	—	—	—	—	—	277
450	500	2	17	77	123	114	51	31	4	6	—	—	—	—	—	—	—	—	425
500	550	2	24	85	147	148	79	25	10	3	—	—	—	—	—	—	—	—	523
550	600	7	34	106	172	177	82	35	8	2	—	—	—	—	—	—	—	—	623
600	650	3	32	107	169	141	61	30	6	1	—	—	—	—	—	—	—	—	550
650	700	2	21	105	177	154	76	18	—	2	—	—	—	—	—	—	—	—	555
700	750	2	27	103	164	115	59	15	4	3	—	—	—	—	—	—	—	—	492
750	800	3	19	67	115	77	38	9	2	1	—	—	—	—	—	—	—	—	331
800	850	2	11	67	86	50	39	4	2	—	—	—	—	—	—	—	—	—	261
850	900	—	12	44	46	40	16	3	1	—	—	—	—	—	—	—	—	—	162
900	950	1	11	23	34	12	8	1	—	—	—	—	—	—	—	—	—	—	90
950	1000	—	4	12	14	15	4	1	—	—	—	—	—	—	—	—	—	—	50
1000	1050	—	2	13	15	5	3	2	—	1	—	—	—	—	—	—	—	—	41
1050	1100	—	3	7	6	5	2	—	1	—	—	—	—	—	—	—	—	—	24
1100	1150	—	1	6	1	4	1	—	—	—	—	—	—	—	—	—	—	—	13
1150	1200	—	1	1	1	1	1	—	—	—	—	—	—	—	—	—	—	—	5
1200	& over	—	4	10	3	1	—	—	—	—	—	—	—	—	—	—	—	—	18
Totals		26	240	896	1417	1194	613	214	47	24									4671

TABLE II. 1911 *Data. Quantity and Weeks* (Diagrams XVIII and XX)

Length of Lactation Period in Weeks

Quantity of milk in gallons Above	Up to	10—12	13—15	16—18	19—21	22—24	25—27	28—30	31—33	34—36	37—39	40—42	43—45	46—48	49—51	52—54	55—57	58—60	61—63	64—66	67—69	70—72	Totals
	200	1	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	2
200	250	—	1	3	4	2	—	2	—	—	—	—	—	—	—	—	—	—	—	—	—	—	12
250	300	—	—	1	1	5	7	2	2	1	—	—	—	—	—	—	—	—	—	—	—	—	19
300	350	—	1	1	4	12	7	20	3	6	2	—	—	—	—	—	—	—	—	—	—	—	56
350	400	—	—	—	5	5	23	34	34	17	8	4	2	—	—	—	—	—	—	—	—	—	132
400	450	—	—	—	1	10	23	54	73	53	47	5	3	1	—	—	—	—	—	—	—	—	270
450	500	—	—	1	1	5	21	89	99	117	50	20	7	3	2	—	—	—	—	—	—	—	415
500	550	—	—	—	1	1	20	65	126	143	91	49	18	4	1	2	—	—	—	—	—	—	521
550	600	—	—	—	—	1	6	56	117	171	137	88	31	7	5	—	1	—	—	—	—	—	620
600	650	—	—	—	—	—	7	28	97	143	145	72	27	17	7	—	—	—	—	—	—	—	543
650	700	—	—	—	1	—	6	23	78	132	141	84	40	28	11	2	1	—	1	—	—	—	548
700	750	—	—	—	—	—	3	9	43	106	120	96	54	25	9	8	2	—	—	—	—	—	475
750	800	—	—	—	—	—	1	2	29	62	84	80	34	15	11	4	2	—	—	—	1	—	325
800	850	—	—	—	—	—	—	5	14	34	68	59	34	19	11	7	—	2	1	—	1	—	255
850	900	—	—	—	—	—	—	1	6	18	44	34	17	23	6	2	2	2	1	—	—	—	156
900	950	—	—	—	—	—	—	—	4	9	20	19	12	10	4	2	—	—	1	—	—	3	84
950	1000	—	—	—	—	—	—	—	1	4	11	13	9	6	6	4	1	—	1	1	—	—	57
1000	1050	—	—	—	—	—	—	—	—	3	3	9	7	7	5	4	—	—	1	2	—	—	41
1050	1100	—	—	—	—	—	—	—	—	2	6	4	4	2	2	—	1	—	1	1	—	1	24
1100	1150	—	—	—	—	—	—	—	—	1	—	2	3	—	3	1	1	—	1	—	—	1	13
1150	1200	—	—	—	—	—	—	1	—	—	1	—	—	1	1	—	—	—	—	—	—	1	5
1200	1250	—	—	—	—	—	—	—	—	—	1	1	—	1	—	1	—	—	—	1	1	1	7
1250	& over	—	—	—	—	—	—	—	—	—	—	—	1	—	1	1	1	—	1	1	—	5	11
Totals		1	2	7	18	41	124	391	726	1022	979	639	303	169	85	38	12	4	9	6	3	12	4591

TABLE III. 1911 *Data. Quantity and Age* (Diagram XIV)

Quantity of milk in gallons Above	Up to	Age in Years 2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	Totals
	200	—	1	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	2
200	250	2	4	3	2	1	—	—	—	—	—	—	—	—	—	—	—	—	12
250	300	5	10	2	2	—	—	—	—	—	—	—	—	—	—	—	—	—	19
300	350	6	23	16	4	2	2	1	—	1	—	—	—	—	—	—	—	1	56
350	400	14	57	29	14	4	5	2	—	5	1	—	1	—	—	—	—	—	132
400	450	7	98	79	32	19	9	9	2	7	2	2	2	1	1	—	—	—	270
450	500	20	120	107	63	34	23	14	6	9	8	4	3	—	1	1	1	1	415
500	550	15	133	112	89	45	43	26	18	14	12	4	4	—	2	3	1	—	521
550	600	15	157	97	102	64	60	48	24	21	14	8	3	6	1	—	—	—	620
600	650	21	107	96	81	65	48	41	27	25	6	7	11	6	1	—	1	—	543
650	700	10	97	74	70	87	56	49	35	35	12	17	5	—	—	—	—	1	548
700	750	11	56	53	69	70	67	47	35	28	17	14	2	1	4	1	—	—	475
750	800	4	39	32	45	34	51	33	29	27	14	16	1	—	—	—	—	—	325
800	850	6	27	19	25	43	32	31	27	20	9	8	3	3	1	1	—	—	255
850	900	1	11	8	17	26	24	16	19	14	9	3	4	1	2	—	1	—	156
900	950	3	5	8	9	10	15	9	9	7	4	5	—	—	—	—	—	—	84
950	1000	—	2	5	10	6	8	7	4	7	4	3	—	—	1	—	—	—	57
1000	1050	1	3	6	4	6	2	7	5	4	—	2	—	—	—	1	—	—	41
1050	1100	—	5	2	—	1	1	6	5	—	—	1	—	3	—	—	—	—	24
1100	1150	—	—	—	3	2	2	—	2	3	—	—	—	1	—	—	—	—	13
1150	1200	—	—	—	—	1	—	1	—	2	—	1	—	—	—	—	—	—	5
1200	& over	—	—	2	3	1	1	3	3	3	2	—	—	—	—	—	—	—	18
Totals		141	955	750	644	521	449	350	251	232	114	95	39	22	14	7	4	3	4591

TABLE IV. 1911 *Data. Age and Percentage* (Diagram XVI)

Percentage butter fat Above	Up to	Age in Years 2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	Totals
2·75	3·00	—	2	4	4	3	2	4	—	1	1	1	2	—	—	—	—	—	24
3·00	3·25	3	13	32	28	29	31	30	11	17	11	11	6	3	—	2	—	1	228
3·25	3·50	12	105	130	134	108	103	83	55	58	40	28	7	4	3	2	1	—	873
3·50	3·75	36	253	226	203	171	125	117	95	88	31	30	14	9	5	—	3	—	1406
3·75	4·00	41	285	201	168	135	118	70	58	46	23	15	7	2	2	1	—	1	1173
4·00	4·25	28	187	96	73	57	54	33	26	17	6	10	3	4	4	—	—	1	599
4·25	4·50	15	77	48	30	13	12	12	5	3	1	—	—	—	—	—	—	—	216
4·50	4·75	4	24	7	3	3	2	—	1	2	1	—	—	—	—	1	—	—	48
4·75	& over	2	9	6	1	2	2	1	—	—	—	—	—	—	—	1	—	—	24
Totals		141	955	750	644	521	449	350	251	232	114	95	39	22	14	7	4	3	4591

TABLE V. 1911 *Data. Age and Yield of Butter Fat* (Diagram XXVI)

Total yield of butter fat in pounds Above	Up to	Age in Years 2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	Totals
40	60	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
60	80	—	1	2	—	—	—	—	1	—	—	—	—	—	—	—	—	—	4
80	100	2	3	1	2	1	—	—	—	—	—	—	—	—	—	—	—	—	9
100	120	4	16	9	2	2	2	1	—	1	—	—	—	—	—	—	—	—	37
120	140	9	35	24	12	3	2	3	—	2	2	1	2	—	—	—	—	1	96
140	160	12	71	63	26	21	11	6	2	9	3	2	2	—	1	—	—	1	230
160	180	15	98	91	66	39	21	20	7	12	8	5	2	2	2	2	1	—	391
180	200	15	118	123	84	37	59	29	20	18	14	6	7	3	—	—	1	—	534
200	220	18	159	117	105	71	41	41	20	24	15	9	4	1	2	1	1	—	629
220	240	17	158	92	81	68	61	52	31	29	6	12	10	6	1	—	—	—	624
240	260	18	114	81	87	82	49	44	41	34	15	16	2	2	1	2	—	—	588
260	280	11	71	53	66	56	73	55	37	27	18	11	3	1	1	—	—	1	484
280	300	6	46	32	45	60	57	31	27	26	9	12	3	2	2	—	—	—	358
300	320	7	27	16	20	31	29	31	22	14	10	7	—	—	2	—	1	—	217
320	340	4	14	18	23	20	23	15	16	12	8	8	3	1	—	2	—	—	167
340	360	1	9	13	12	13	10	10	5	9	3	1	1	—	1	—	—	—	88
360	380	2	6	7	1	10	8	5	9	7	1	2	—	—	—	—	—	—	58
380	400	—	5	1	2	4	1	1	8	1	—	1	—	2	1	—	—	—	27
400	420	—	1	2	5	—	1	—	—	1	—	—	—	—	—	—	—	—	10
420	440	—	1	1	4	2	—	2	2	1	1	1	—	1	—	—	—	—	16
440	460	—	—	1	—	—	1	1	1	2	—	—	—	1	—	—	—	—	7
460	480	—	1	3	1	1	—	3	2	3	1	1	—	—	—	—	—	—	16
Totals		141	955	750	644	521	449	350	251	232	114	95	39	22	14	7	4	3	4591

Age in Years

Length of lactation period in weeks	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	Totals
10—12	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
13—15	—	—	—	1	—	1	—	—	—	—	—	—	—	—	—	—	—	2
16—18	—	4	1	—	—	—	—	1	—	1	—	—	—	—	—	—	—	7
19—21	—	1	6	5	1	—	1	1	1	1	—	1	—	—	—	—	—	18
22—24	5	11	12	3	6	1	1	—	1	—	—	—	—	—	—	—	1	41
25—27	5	21	21	25	13	12	6	7	6	2	4	—	1	1	—	—	—	124
28—30	16	51	74	55	41	35	27	25	25	17	8	9	3	2	1	2	—	391
31—33	24	108	136	101	90	73	64	36	44	15	16	11	3	2	2	1	—	726
34—36	22	173	182	157	119	111	88	55	48	29	21	9	5	2	—	—	1	1022
37—39	20	181	152	143	129	107	78	64	39	24	28	6	3	2	1	1	1	979
40—42	16	181	87	87	64	56	47	36	33	16	7	2	6	—	1	—	—	639
43—45	12	100	33	31	26	31	25	10	21	5	5	1	—	2	1	—	—	303
46—48	8	63	25	20	20	10	3	9	4	2	3	—	—	1	1	—	—	169
49—51	8	35	8	7	5	5	6	3	4	—	3	—	—	1	—	—	—	85
52—54	3	10	8	4	2	6	1	2	1	1	—	—	—	—	—	—	—	38
55—57	1	3	2	1	2	1	1	—	—	—	—	—	1	—	—	—	—	12
Totals	140	943	747	640	518	449	348	249	227	113	95	39	22	13	7	4	3	4557

TABLE VII. 1911 *Data. Weeks and Percentage of Fat* (Diagram XXII)

Percentage of Fat

Length of lactation period in weeks (Above / Up to)	*2·75 3·00*	*3·00 3·25*	*3·25 3·50*	*3·50 3·75*	*3·75 4·00*	*4·00 4·25*	*4·25 4·50*	*4·50 4·75*	*4·75* and over	Totals
10—12	—	—	1	—	—	—	—	—	—	1
13—15	—	—	1	1	—	—	—	—	—	2
16—18	—	—	2	2	1	2	—	—	—	7
19—21	—	1	5	7	3	2	—	—	—	18
22—24	—	3	4	15	14	4	1	—	—	41
25—27	1	4	12	40	33	23	7	3	1	124
28—30	3	29	79	128	88	37	22	3	2	391
31—33	7	42	160	211	193	75	32	6	—	726
34—36	3	64	211	326	243	123	39	9	4	1022
37—39	3	42	179	325	242	127	50	6	5	979
40—42	3	22	108	178	181	103	30	9	5	639
43—45	1	17	51	76	91	42	15	7	3	303
46—48	2	2	27	50	37	36	10	3	2	169
49—51	1	3	19	25	18	12	7	—	—	85
52—54	—	—	7	8	16	4	2	—	1	38
55—57	—	—	3	1	3	2	1	2	—	12
Totals	24	229	869	1393	1163	592	216	48	23	4557

TABLE VIII. 1912 *Data. Quantity and Percentage* (Diagrams IX and XI)

Percentage of Fat

Quantity of milk in gallons (Above / Up to)	*2·25*	*2·25 2·50*	*2·50 2·75*	*2·75 3·00*	*3·00 3·25*	*3·25 3·50*	*3·50 3·75*	*3·75 4·00*	*4·00 4·25*	*4·25 4·50*	*4·50 4·75*	*4·75* & over	Totals
200	—	—	—	—	1	—	2	2	2	1	1	—	9
200 250	—	—	—	—	—	3	4	8	4	3	2	1	25
250 300	—	—	—	—	4	2	10	13	15	7	4	2	57
300 350	—	—	—	2	8	10	22	30	13	9	3	3	100
350 400	—	—	1	—	9	26	46	50	27	11	6	1	177
400 450	—	—	—	2	19	41	82	65	54	30	7	2	302
450 500	—	—	—	2	22	65	133	143	71	45	16	6	503
500 550	—	—	—	10	26	106	163	173	105	38	8	8	637
550 600	—	—	—	7	49	126	212	214	132	44	19	3	806
600 650	—	—	1	6	47	160	208	230	124	36	14	3	829
650 700	1	—	1	4	44	130	201	232	105	35	9	—	762
700 750	—	—	—	5	47	121	221	185	79	24	5	1	688
750 800	—	—	2	5	25	120	169	132	51	19	3	1	527
800 850	—	—	—	3	23	83	146	135	39	17	2	—	448
850 900	—	—	—	1	18	59	85	54	23	3	3	2	248
900 950	—	—	—	2	13	43	61	48	17	5	—	—	189
950 1000	—	—	—	2	6	30	28	34	4	2	2	—	108
1000 1050	—	—	—	1	7	29	33	11	6	4	—	—	91
1050 1100	—	—	—	—	3	12	14	6	5	3	—	—	43
1100 1150	—	—	—	—	3	13	9	12	2	1	—	—	40
1150 1200	—	—	—	—	—	5	8	5	1	—	—	—	19
1200 & over	—	—	—	—	6	14	12	4	2	1	—	1	40
Totals	1	—	5	52	380	1198	1869	1786	881	338	104	34	6648

TABLE IX

1912 *Data. Quantity and Weeks* (Diagrams XIX and XXI)

Length of Lactation Period in Weeks

Quantity of milk in gallons *Above*	*Up to*	*7— 9*	*10—12*	*13—15*	*16—18*	*19—21*	*22—24*	*25—27*	*28—30*	*31—33*	*34—36*	*37—39*	*40—42*	*43—45*	*46—48*	*49—51*	*52—54*	*55—57*	*58—60*	*61—63*	*64—66*	*67—69*	*70—72*	*73—75*	*76—78*	*79 and over*	Totals
	200	—	—	1	5	3	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	9
200	*250*	1	—	3	6	4	5	4	2	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	25
250	*300*	—	—	—	3	6	17	14	12	2	3	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	57
300	*350*	—	—	—	6	10	21	18	24	8	10	1	1	1	—	—	—	—	—	—	—	—	—	—	—	—	100
350	*400*	—	—	—	7	9	12	31	45	37	28	7	1	—	—	—	—	—	—	—	—	—	—	—	—	—	177
400	*450*	—	—	—	1	4	8	47	50	83	63	31	11	4	—	—	—	—	—	—	—	—	—	—	—	—	302
450	*500*	—	—	—	—	2	5	32	97	119	128	66	37	16	1	—	—	—	—	—	—	—	—	—	—	—	503
500	*550*	—	—	—	—	—	4	20	69	146	186	125	56	19	12	—	—	—	—	—	—	—	—	—	—	—	637
550	*600*	—	—	—	—	2	1	16	52	126	241	189	110	45	17	7	—	—	—	—	—	—	—	—	—	—	806
600	*650*	—	—	—	—	—	—	6	39	116	225	211	131	57	28	9	3	3	1	—	—	—	—	—	—	—	829
650	*700*	—	—	—	—	—	1	3	19	85	173	215	148	66	36	12	4	—	—	—	—	—	—	—	—	—	762
700	*750*	—	—	—	—	—	1	2	16	43	160	185	152	67	32	16	9	5	—	—	—	—	—	—	—	—	688
750	*800*	—	—	—	—	—	1	2	10	36	95	136	102	60	38	30	10	3	4	—	—	—	—	—	—	—	527
800	*850*	—	—	—	—	—	—	2	7	14	70	111	101	60	37	20	10	9	2	3	1	—	1	—	—	—	448
850	*900*	—	—	—	—	—	—	—	2	11	30	45	55	41	22	20	9	3	2	3	1	3	—	—	—	1	248
900	*950*	—	—	—	—	—	—	—	—	1	14	35	45	40	18	17	6	5	2	1	3	—	1	1	—	—	189
950	*1000*	—	—	—	—	—	—	—	—	2	10	14	26	19	9	13	7	3	4	—	—	1	—	—	—	—	108
1000	*1050*	—	—	—	—	—	—	—	—	2	3	16	15	24	18	4	2	5	—	—	1	1	—	—	—	—	91
1050	*1100*	—	—	—	—	—	—	—	—	—	2	5	10	2	10	5	2	2	1	3	—	1	—	—	—	—	43
1100	*1150*	—	—	—	—	—	—	—	—	—	1	5	11	4	6	4	5	—	2	1	1	—	—	—	—	—	40
1150	*1200*	—	—	—	—	—	—	—	—	—	1	1	1	4	5	3	2	—	1	—	—	—	—	—	1	—	19
1200	*1250*	—	—	—	—	—	—	—	—	—	—	—	1	2	—	3	2	1	1	1	—	—	—	—	—	1	12
1250 & over		—	—	—	—	—	—	—	—	—	—	1	2	3	5	2	4	2	1	1	1	3	—	—	1	2	28
Totals		1	—	4	28	40	76	197	444	831	1443	1399	1016	534	294	165	75	41	21	13	8	9	2	1	2	4	6648

TABLE X. 1912 *Data. Quantity and Age* (Diagram XV)

Quantity of milk in gallons Above	Up to	Age in Years 2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	Totals
	200	3	2	3	—	—	1	—	—	—	—	—	—	—	—	—	—	—	9
200	250	4	12	7	2	—	—	—	—	—	—	—	—	—	—	—	—	—	25
250	300	10	29	11	3	4	—	—	—	—	—	—	—	—	—	—	—	—	57
300	350	14	37	23	8	7	5	2	1	1	—	—	1	1	—	—	—	—	100
350	400	21	56	45	23	12	6	6	4	1	3	—	—	—	—	—	—	—	177
400	450	15	114	74	35	24	16	4	10	4	2	1	2	1	—	—	—	—	302
450	500	23	171	129	69	41	25	21	10	4	4	3	2	1	—	—	—	—	503
500	550	33	175	125	89	62	44	44	25	20	6	5	3	4	1	1	—	—	637
550	600	29	239	137	123	88	54	53	37	15	14	9	4	2	1	—	1	—	806
600	650	22	194	136	111	120	64	72	41	27	16	11	9	4	—	—	—	2	829
650	700	19	134	100	131	112	83	62	40	37	21	12	6	2	2	1	—	—	762
700	750	16	100	94	89	86	77	76	58	40	26	15	6	1	1	1	—	2	688
750	800	13	75	65	71	70	61	63	44	29	19	8	5	1	2	—	—	1	527
800	850	10	53	41	62	51	57	62	40	29	15	10	11	6	—	1	—	—	448
850	900	3	34	28	29	43	18	34	17	19	12	6	3	2	—	—	—	—	248
900	950	2	15	17	23	25	31	18	22	20	8	4	1	—	1	2	—	—	189
950	1000	2	12	4	11	14	16	16	12	8	5	3	2	2	1	—	—	—	108
1000	1050	1	—	6	14	12	9	16	12	5	11	2	2	1	—	—	—	—	91
1050	1100	1	2	2	5	6	9	6	5	2	3	2	—	—	—	—	—	—	43
1100	1150	3	1	4	4	4	5	2	5	4	1	3	—	3	1	—	—	—	40
1150	1200	—	—	1	1	1	3	2	2	7	1	1	—	—	—	—	—	—	19
1200	& over	—	—	1	7	6	6	4	6	6	1	1	1	—	1	—	—	—	40
Totals		244	1455	1053	910	788	590	563	391	278	168	96	58	31	11	6	1	5	6648

TABLE XI. 1912 *Data. Age and Percentage* (Diagram XVII)

Percentage butter fat Above	Up to	Age in Years 2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	Totals
	2·25	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	1
2·25	2·50	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
2·50	2·75	—	1	—	3	—	—	—	1	—	—	—	—	—	—	—	—	—	5
2·75	3·00	—	4	10	5	6	6	8	6	3	1	2	1	—	—	—	—	—	52
3·00	3·25	9	29	47	55	50	46	40	47	23	13	10	8	2	—	—	—	1	380
3·25	3·50	36	147	154	161	161	134	141	92	81	43	23	16	6	2	1	—	—	1198
3·50	3·75	45	336	309	269	233	186	178	107	84	55	33	17	8	4	3	—	2	1869
3·75	4·00	78	441	298	248	210	133	129	98	64	35	21	12	12	4	1	—	2	1786
4·00	4·25	39	311	155	113	97	52	44	26	14	16	7	3	3	1	—	—	—	881
4·25	4·50	24	136	52	44	23	22	15	10	7	3	—	1	—	—	1	—	—	338
4·50	4·75	11	38	20	10	7	9	4	3	1	1	—	—	—	—	—	—	—	104
4·75	& over	2	12	8	1	1	2	4	1	1	1	—	—	—	—	—	1	—	34
Totals		244	1455	1053	910	788	590	563	391	278	168	96	58	31	11	6	1	5	6648

TABLE XII. 1912 *Data. Age and Yield of Butter Fat* (Diagram XXVII)

Total yield of butter fat in pounds Above	Up to	Age in Years 2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	Totals
60	80	3	2	4	1	—	1	—	—	—	—	—	—	—	—	—	—	—	11
80	100	5	13	12	2	2	—	—	—	—	—	—	—	—	—	—	—	—	34
100	120	9	31	10	7	5	3	—	1	1	—	—	1	—	—	—	—	—	68
120	140	16	54	39	28	14	9	6	4	1	4	—	—	1	—	—	—	—	176
140	160	19	78	67	34	22	12	7	12	5	—	2	2	1	—	—	—	—	261
160	180	30	110	115	60	40	33	28	16	13	5	3	2	4	1	—	—	—	460
180	200	19	167	137	85	78	36	35	26	16	10	8	2	1	—	1	—	2	623
200	220	39	251	135	124	92	64	50	38	17	9	10	7	2	—	—	—	—	838
220	240	28	214	140	106	93	68	74	43	25	20	15	8	3	1	1	—	—	839
240	260	20	179	103	123	102	79	83	52	50	25	12	8	2	2	—	—	—	840
260	280	20	122	107	107	108	88	74	55	36	21	10	8	5	1	—	—	1	763
280	300	12	97	66	76	83	60	71	32	29	25	9	9	1	1	2	1	2	576
300	320	8	63	47	54	56	41	51	39	22	10	9	4	5	1	1	—	—	411
320	340	3	32	33	40	28	35	37	24	20	12	6	3	—	—	—	—	—	273
340	360	4	24	14	26	26	15	21	16	17	12	2	1	2	—	—	—	—	180
360	380	4	8	11	14	18	16	8	10	11	7	4	—	1	2	—	—	—	114
380	400	1	5	6	7	6	13	10	6	4	1	2	1	—	—	1	—	—	63
400	420	1	4	1	4	4	7	1	7	3	4	2	1	1	—	—	—	—	40
420	440	3	—	2	3	7	4	1	4	4	3	—	—	—	1	—	—	—	32
440	460	—	1	2	4	2	1	3	1	2	—	2	—	1	—	—	—	—	19
460	& over	—	—	2	5	2	5	3	5	2	—	—	1	1	1	—	—	—	27
Totals		244	1455	1053	910	788	590	563	391	278	168	96	58	31	11	6	1	5	6648

TABLE XIII. 1912 *Data. Age and Weeks* (Diagram XXV)

Age in Years

Length of lactation period in weeks	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*	*13*	*14*	*15*	*16*	*17*	*18*	Totals
7— 9	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
10—12	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
13—15	1	1	2	—	—	—	—	—	—	—	—	—	—	—	—	—	—	4
16—18	—	3	11	5	2	5	1	—	—	1	—	—	—	—	—	—	—	28
19—21	8	6	15	4	1	2	2	—	1	1	—	—	—	—	—	—	—	40
22—24	5	21	12	14	8	3	3	5	1	1	1	2	—	—	—	—	—	76
25—27	8	50	37	35	18	13	12	13	4	4	2	—	1	—	—	—	—	197
28—30	24	71	87	58	57	48	34	21	20	12	2	5	3	1	—	—	1	444
31—33	33	135	137	130	108	79	69	46	41	26	9	12	5	1	—	—	—	831
34—36	28	251	252	191	194	142	139	97	65	34	25	15	4	3	2	—	1	1443
37—39	23	243	232	230	174	120	136	101	65	27	24	11	6	2	2	1	2	1399
40—42	39	277	129	114	123	88	82	60	38	28	20	5	9	2	1	—	1	1016
43—45	25	177	65	59	47	48	47	21	17	16	5	5	1	1	—	—	—	534
46—48	23	102	34	30	33	18	16	11	12	10	3	1	—	1	—	—	—	294
49—51	11	67	16	21	12	11	11	8	3	3	2	—	—	—	—	—	—	165
52—54	7	19	11	6	6	8	4	4	5	—	2	1	1	—	1	—	—	75
55—57	3	14	3	8	3	1	4	1	—	3	—	1	—	1	—	—	—	41
58—60	—	11	2	—	—	1	2	—	3	—	1	—	1	—	—	—	—	21
61—63	1	3	2	2	1	2	1	—	—	1	—	—	—	—	—	—	—	13
64—66	2	1	2	1	1	—	—	—	1	—	—	—	—	—	—	—	—	8
67—69	1	2	1	—	—	1	—	1	2	1	—	—	—	—	—	—	—	9
70—72	1	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	2
73—75	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
76—78	—	—	1	1	—	—	—	—	—	—	—	—	—	—	—	—	—	2
79 and over	1	—	—	1	—	—	—	2	—	—	—	—	—	—	—	—	—	4
Totals	244	1455	1053	910	788	590	563	391	278	168	96	58	31	11	6	1	5	6648

TABLE XIV. 1912 *Data. Weeks and Percentage* (Diagram XXIII)

Percentage of Fat

Length of lactation period in weeks	*Upto 2·25*	*2·25 2·50*	*2·50 2·75*	*2·75 3·00*	*3·00 3·25*	*3·25 3·50*	*3·50 3·75*	*3·75 4·00*	*4·00 4·25*	*4·25 4·50*	*4·50 4·75*	*4·75* and over	Totals
7— 9	—	—	—	—	—	—	—	—	—	—	—	1	1
10—12	—	—	—	—	—	—	—	—	—	—	—	—	—
13—15	—	—	—	—	—	—	—	3	1	—	—	—	4
16—18	—	—	—	1	5	3	7	6	4	1	1	—	28
19—21	—	—	—	1	1	8	9	13	6	—	1	1	40
22—24	—	—	—	—	10	11	21	17	7	7	1	2	76
25—27	—	—	—	3	14	28	59	44	28	14	6	1	197
28—30	—	—	1	5	39	93	106	102	59	27	9	3	444
31—33	—	—	—	9	50	158	235	208	110	48	10	3	831
34—36	—	—	—	17	86	287	423	369	170	60	24	7	1443
37—39	1	—	3	7	87	249	396	410	166	63	14	3	1399
40—42	—	—	1	7	43	170	314	279	135	50	14	3	1016
43—45	—	—	—	—	21	78	141	154	91	35	10	4	534
46—48	—	—	—	1	9	49	82	78	53	15	6	1	294
49—51	—	—	—	—	10	32	35	54	21	8	4	1	165
52—54	—	—	—	—	2	16	16	24	12	2	3	—	75
55—57	—	—	—	—	1	6	11	11	7	4	—	1	41
58—60	—	—	—	—	1	4	5	4	4	1	1	1	21
61—63	—	—	—	1	1	1	3	5	1	1	—	—	13
64—66	—	—	—	—	—	1	2	1	3	1	—	—	8
67—69	—	—	—	—	—	3	—	1	3	1	—	1	9
70—72	—	—	—	—	—	1	—	1	—	—	—	—	2
73 and over	—	—	—	—	—	—	4	2	—	—	—	1	7
Totals	1	—	5	52	380	1198	1869	1786	881	338	104	34	6648

TABLE XV. *Distribution of Age*
(Diagrams I and II)

Age in years	1911 data		1912 data	
	Theory	Observation	Theory	Observation
2·5	222·0	141	453·8	244
3·5	839·2	955	1113·7	1455
4·5	772·6	750	1113·3	1053
5·5	662·3	644	987·1	910
6·5	546·8	521	821·5	788
7·5	438·0	449	652·6	590
8·5	340·8	350	486·2	563
9·5	257·0	251	364·2	391
10·5	187·1	232	255·3	278
11·5	130·7	114	170·5	168
12·5	86·6	95	107·5	96
13·5	53·8	39	63·2	58
14·5	30·5	22	34·0	31
15·5	15·3	14	16·2	11
16·5	6·3	7	6·5	6
17·5	1·8	4	2·0	1
18·5	·2	3	·4	3
19·5	—	—	—	1
20·5	—	—	—	1
	4591·0	4591	6648·0	6648

TABLE XVI
Distribution of Quantity

Quantity in gallons	1911 data		1912 data	
	Theory	Observation	Theory	Observation
25	—	—	·1	1
75	·1	—	·5	1
125	·3	—	1·7	2
175	1·4	2	5·8	5
225	6·5	12	17·3	25
275	23·1	19	45·0	57
325	64·7	58	101·2	100
375	145·3	140	197·7	177
425	267·3	277	336·1	302
475	409·9	425	500·0	503
525	535·0	523	655·7	637
575	606·7	623	765·0	806
625	606·7	550	802·3	829
675	546·1	555	764·1	762
725	448·6	492	668·0	688
775	341·0	331	541·3	527
825	242·8	261	410·9	448
875	163·6	162	294·6	248
925	105·3	90	201·2	189
975	65·2	50	131·8	108
1025	39·1	41	83·4	91
1075	22·9	24	51·3	43
1125	13·1	13	30·8	40
1175	7·4	5	18·1	19
1225	4·1	9	10·5	21
1275	2·3	5	6·0	12
1325	1·2	3	3·4	6
1375	·7	1	1·9	1
1425	·4	—	1·1	—
1475	·2	—	·6	—
1525	—	—	·3	—
1575	—	—	·2	—
1625	—	—	·1	—
	4671·0	4671	6648·0	6648

TABLE XVII. *Distribution of Percentage of Butter Fat* (Diagrams VI and VII)

Percentage butter fat	1911 data		1912 data	
	Theory	Observation	Theory	Observation
2·125	—	—	—	1
2·375	—	—	·2	0
2·625	·5	—	4·7	5
2·875	22·4	26	59·0	52
3·125	245·9	240	376·4	380
3·375	894·5	896	1186·3	1198
3·625	1418·3	1417	1911·2	1869
3·875	1188·8	1194	1707·6	1786
4·125	611·0	613	932·5	881
4·375	216·4	214	346·1	338
4·625	57·9	47	96·6	104
4·875	12·5	24	22·1	34
5·125	2·8	—	5·3	—
	4671·0	4671	6648·0	6648

TABLE XVIII
Distribution of Length of Lactation Period

Number of weeks in milk	1911 data		1912 data	
	Theory	Observation	Theory	Observation
2	—	—	·3	—
5	—	—	·4	—
8	·1	—	·8	1
11	·3	1	1·7	0
14	1·1	2	3·2	4
17	3·9	7	10·2	28
20	14·3	18	27·9	40
23	49·5	41	77·1	76
26	152·0	124	207·3	197
29	381·2	391	496·7	444
32	723·2	726	949·6	831
35	983·8	1022	1323·8	1443
38	947·4	979	1314·5	1399
41	665·8	639	975·3	1016
44	362·7	303	591·7	534
47	164·7	169	320·4	294
50	66·6	85	165·1	165
53	25·4	38	84·4	75
56	9·4	12	43·3	41
59	3·5	—	23·2	21
62	1·3	—	13·1	13
65	·5	—	7·2	8
68	·2	—	4·2	9
71	·1	—	2·5	2
74	—	—	1·5	1
77	—	—	1·0	2
80	—	—	·6	2
83	—	—	·4	1
86	—	—	·6	1
	4557·0	4557	6648·0	6648

TABLE XIX

Distribution of Total Yield of Butter Fat

Yield of butter in pounds	1911 data		1912 data	
	Theory	Observation	Theory	Observation
10	—	—	·1	—
30	·2	—	·6	—
50	1·1	1	2·4	3
70	5·0	4	8·5	8
90	18·6	9	26·1	34
110	54·9	37	68·8	68
130	129·7	96	153·5	176
150	248·3	230	291·1	261
170	390·4	391	471·6	460
190	522·2	534	658·3	623
210	598·0	629	800·5	838
230	601·8	624	858·6	839
250	543·9	588	823·1	840
270	446·3	484	714·6	763
290	340·2	358	568·8	576
310	243·6	217	421·0	411
330	165·9	167	292·4	273
350	108·4	88	192·6	180
370	68·0	58	121·4	114
390	42·3	27	73·8	63
410	25·6	10	43·4	40
430	15·3	16	25·1	32
450	9·0	7	14·2	19
470	5·3	6	7·9	12
490	3·1	4	4·4	8
510	1·8	3	2·4	4
530	1·0	2	1·3	2
550	·6	1	·7	1
570	·3	—	·4	—
590	·2	—	·2	—
610	—	—	·1	—
630	—	—	·1	—
	4591·0	4591	6648·0	6648

TABLE XX (*a*)*

Distribution of Quantity of Milk for 2 and 3 Years of Age

Quantity in gallons	1911 and 1912 data combined			
	2·5 years		3·5 years	
	Theory	Observation	Theory	Observation
25	·1	—	—	—
75	·3	—	·2	—
125	1·0	1	·9	1
175	2·6	2	3·5	2
225	5·9	6	11·3	16
275	11·4	15	30·3	39
325	19·3	20	68·5	60
375	28·5	35	130·0	113
425	37·3	22	208·4	212
475	43·5	43	284·5	291
525	45·7	48	333·0	308
575	43·7	44	339·3	396
625	38·4	43	304·5	301
675	31·4	29	243·8	231
725	24·1	27	176·4	156
775	17·6	17	117·3	114
825	12·2	16	71·8	80
875	8·2	4	41·3	45
925	5·3	5	22·5	20
975	3·3	2	11·7	14
1025	2·1	2	5·9	3
1075	1·3	1	2·8	7
1125	·8	1	1·3	1
1175	·4	1	·6	—
1225	·3	1	·3	—
1275	·2	—	·1	—
1325	·1	—	—	—
1375	—	—	—	—
	385·00	385	2410·2	2410

TABLE XX (*b*)*

Distribution of Quantity of Milk for 4 and 5 Years of Age

Quantity in gallons	1911 and 1912 data combined			
	4·5 years		5·5 years	
	Theory	Observation	Theory	Observation
25	—	—	—	—
75	·1	—	—	—
125	·3	1	·1	—
175	1·5	2	·3	1
225	5·6	10	1·2	3
275	17·2	13	4·7	5
325	43·0	39	14·2	12
375	87·9	74	36·5	37
425	147·8	153	76·1	67
475	206·7	236	129·0	132
525	244·4	237	179·4	178
575	249·9	234	210·1	225
625	225·3	232	212·1	192
675	182·8	174	189·4	201
725	136·3	147	153·3	158
775	94·8	97	114·8	116
825	62·3	60	81·1	87
875	39·3	36	54·8	46
925	23·9	25	35·8	32
975	14·2	9	22·9	21
1025	8·3	12	14·4	18
1075	4·8	4	9·0	5
1125	2·8	4	5·6	7
1175	1·6	1	3·5	1
1225	·9	2	2·2	4
1275	·5	1	1·4	3
1325	·3	—	·8	2
1375	·2	—	·5	1
1425	·1	—	·3	—
1475	—	—	·2	—
1525	—	—	·2	—
	1802·8	1803	1553·9	1554

* See Table VI of text and Diagrams III and IV.

TABLE XX (*c*)*

Distribution of Quantity of Milk for 6 *and* 7 *Years of Age*

Quantity in gallons	1911 and 1912 data combined			
	6·5 years		7·5 years	
	Theory	Observation	Theory	Observation
75	—	—	—	—
125	—	—	·1	—
175	·2	—	·2	1
225	.6	2	·6	1
275	2·2	3	1·9	2
325	6·9	9	5·2	4
375	18·4	16	12·3	11
425	41·0	43	26·5	25
475	76·7	75	49·4	48
525	120·4	107	79·3	87
575	158·2	152	109·8	114
625	178·7	185	131·7	112
675	176·1	199	138·3	139
725	154·2	156	128·8	144
775	122·3	104	108·1	112
825	89·5	94	83·0	89
875	61·4	69	59·3	42
925	40·1	35	39·9	46
975	25·1	20	25·6	24
1025	15·3	18	15·9	11
1075	9·2	7	9·6	10
1125	5·4	6	5·7	7
1175	3·1	2	3·3	3
1225	1·8	4	1·9	4
1275	1·1	2	1·1	2
1325	.6	1	·6	1
1375	·4	—	·4	—
1425	·2	—	·2	—
1475	·1	—	·1	—
1525	—	—	·1	—
1575	—	—	—	—
	1309·2	1309	1038·9	1039

TABLE XX (*d*)*

Distribution of Quantity of Milk for 8 *and* 9 *Years of Age*

Quantity in gallons	1911 and 1912 data combined			
	8·5 years		9·5 years	
	Theory	Observation	Theory	Observation
175	—	—	—	1
225	—	—	·1	—
275	·3	1	·4	—
325	1·7	2	1·4	1
375	6·4	8	4·3	4
425	17·5	13	10·8	12
475	38·2	35	22·6	16
525	66·4	70	39·2	43
575	95·2	101	57·7	61
625	115·7	113	73·1	68
675	122·2	111	81·0	75
725	114·7	123	80·0	93
775	97·6	96	71·5	73
825	76·3	93	58·9	67
875	55·9	50	45·2	36
925	38·7	27	32·8	31
975	25·6	23	22·6	16
1025	16·3	23	15·1	17
1075	10·0	12	9·7	10
1125	6·0	2	6·1	7
1175	3·6	3	3·8	2
1225	2·1	3	2·3	4
1275	1·2	2	1·4	3
1325	·7	2	·8	2
1375	·4	—	·5	—
1425	·2	—	·3	—
1475	·1	—	·2	—
1525	·1	—	·1	—
1575	—	—	·1	—
1625	—	—	—	—
	913·1	913	642·0	642

TABLE XX (*e*)*

Distribution of Quantity of Milk for 10 *and* 11 *Years of Age and Over*

Quantity in gallons	1911 and 1912 data combined			
	10·5 years		11·5 years & over	
	Theory	Observation	Theory	Observation
175	—	—	—	—
225	—	—	·1	—
275	·2	—	·5	2
325	1·0	2	2·0	2
375	3·5	6	6·1	5
425	9·3	11	14·6	13
475	19·4	13	28·4	29
525	32·6	34	45·9	46
575	46·2	36	63·4	63
625	56·5	52	76·6	74
675	61·2	72	82·2	79
725	60·0	68	79·8	91
775	53·9	56	71·0	67
825	45·2	49	58·7	68
875	35·7	33	45·5	43
925	26·9	27	33·4	25
975	19·4	15	23·4	21
1025	13·5	9	15·8	19
1075	9·1	2	10·3	9
1125	6·0	7	6·5	6
1175	3·9	9	4·0	5
1225	2·5	4	2·4	3
1275	1·6	3	1·4	2
1325	1·0	2	·8	1
1375	·6	—	·5	1
1425	·4	—	·3	—
1475	·2	—	·2	—
1525	·1	—	·1	—
1575	·1	—	·1	—
1625	—	—	—	—
	510·0	510	674·0	674

* See Table VI of text and Diagrams III and IV.

TABLE XXI. *Regression of Quantity on Percentage* (Diagrams VIII and IX)

Quantity in gallons

Percentage of butter fat	1911		1912	
	Theory	Observation	Theory	Observation
2·125	—	—	—	675·00
2·375	—	—	—	—
2·625	—	—	—	645·00
2·875	697·86	628·85	723·74	650·96
3·125	679·24	672·92	703·60	668·03
3·375	660·62	667·86	683·46	691·57
3·625	641·99	641·02	663·32	670·10
3·875	623·37	620·39	643·18	648·71
4·125	604·75	612·19	623·04	615·12
4·375	586·13	570·09	602·90	592·75
4·625	567·51	575·00	582·75	554·33
4·875	548·88	560·42	562·61	532·35

TABLE XXII. *Regression of Percentage on Quantity* (Diagrams X and XI)

Percentage butter fat

Quantity in gallons	1911		1912	
	Theory	Observation	Theory	Observation
175	3·88	3·75	3·90	3·93
225	3·86	3·79	3·88	3·97
275	3·85	3·95	3·86	3·98
325	3·83	3·78	3·85	3·82
375	3·81	3·84	3·83	3·79
425	3·80	3·81	3·81	3·80
475	3·78	3·77	3·80	3·83
525	3·76	3·77	3·78	3·77
575	3·75	3·74	3·76	3·77
625	3·73	3·72	3·75	3·74
675	3·71	3·72	3·73	3·74
725	3·70	3·70	3·72	3·70
775	3·68	3·68	3·70	3·68
825	3·67	3·68	3·68	3·70
875	3·65	3·65	3·67	3·66
925	3·63	3·58	3·65	3·65
975	3·62	3·66	3·63	3·64
1025	3·60	3·66	3·62	3·59
1075	3·58	3·63	3·60	3·67
1125	3·57	3·59	3·58	3·63
1175	3·55	3·63	3·57	3·65
1225	3·53	3·39	3·55	3·56

TABLE XXIII. *Regression of Quantity on Age* (Diagrams XIV and XV)

Quantity in Gallons

Age	1911		Age	1912	
	Theory	Observed		Theory	Observed
2·5	513·04	557·27	2·5	538·05	558·61
3·5	550·12	567·04	3·5	579·94	582·59
4·5	583·12	580·20	4·5	617·31	599·03
5·5	613·20	527·25	5·5	650·14	656·54
6·5	639·19	669·24	6·5	678·45	680·39
7·5	661·48	681·35	7·5	702·24	709·24
8·5	680·08	698·14	8·5	721·49	718·34
9·5	694·98	729·18	9·5	736·22	733·82
10·5	706·18	710·78	10·5	746·42	758·81
11·5	713·68	699·56	11·5	752·09	750·00
12·5	717·49	725·53	12·5	753·24	747·92
13·5 14·7	712·85	664·89	13·5 14·4	742·94	734·38
14·5			14·5		
15·5			15·5		
16·5			16·5		
17·5			17·5		
18·5			18·5		

TABLE XXIV. *Regression of Age on Quantity*

Age in years

Quantity in gallons	1911		1912	
	Theory	Observation	Theory	Observation
625	5·91	5·72	5·70	5·58
675	6·14	6·11	5·93	5·91
725	6·37	6·48	6·15	6·34
775	6·60	6·75	6·37	6·33
825	6·83	6·96	6·59	6·77
875	7·06	7·46	6·81	6·59
925	7·29	7·02	7·04	7·07
975	7·52	7·46	7·26	7·25
1025	7·75	6·98	7·48	7·71
1075	7·98	7·63	7·70	7·21
1125	8·21	7·92	7·92	7·73
1175	8·44	9·20	8·15	8·58
1225	8·67	7·72	8·37	7·85

TABLE XXV. *Regression of Percentage on Age* (Diagrams XVI and XVII)

Percentage of butter fat

Age	1911		Age	1912	
	Theory	Observation		Theory	Observation
2·5	3·89	3·89	*2·5*	3·92	3·86
3·5	3·84	3·86	*3·5*	3·85	3·88
4·5	3·78	3·76	*4·5*	3·79	3·77
5·5	3·74	3·71	*5·5*	3·74	3·72
6·5	3·70	3·69	*6·5*	3·70	3·70
7·5	3·67	3·69	*7·5*	3·66	3·67
8·5	3·64	3·64	*8·5*	3·64	3·65
9·5	3·62	3·68	*9·5*	3·62	3·62
10·5	3·61	3·63	*10·5*	3·61	3·61
11·5	3·61	3·57	*11·5*	3·61	3·64
12·5	3·61	3·58	*12·5*	3·61	3·59
13·5 *14·7* *14·5* *15·5* *16·5* *17·5* *18·5*	3·64	3·62	*13·5* *14·4* *14·5* *15·5* *16·5* *17·5* *18·5*	3·64	3·64

TABLE XXVI. *Regression of Age on Percentage*

Percentage of butter fat	1911		1912	
	Theory	Observation	Theory	Observation
2·125	—	—	9·08	5·50
2·375	—	—	8·63	—
2·625	—	—	8·19	5·90
2·875	7·92	7·38	7·74	7·25
3·125	7·45	7·68	7·29	7·48
3·375	6·98	6·98	6·84	6·92
3·625	6·51	6·51	6·39	6·43
3·875	6·04	5·93	5·95	5·95
4·125	5·57	5·69	5·50	5·29
4·375	5·10	4·88	5·05	4·99
4·625	4·63	4·90	4·60	4·84
4·875	4·15	5·08	4·15	5·68

TABLE XXVII. *Regression of Quantity on Weeks in Milk* (Diagrams XVIII and XIX)

Quantity in gallons

Weeks in milk	1911		1912	
	Theory	Observation	Theory	Observation
8	—	—	192·58	225·00
11	259·80	175·00	239·66	—
14	303·38	275·00	286·75	212·50
17	346·96	275·00	333·84	287·50
20	390·55	358·33	380·93	337·50
23	434·13	373·78	428·02	365·13
26	477·71	466·94	475·11	444·29
29	521·29	509·14	522·20	508·90
32	564·88	570·45	569·28	569·16
35	608·46	612·38	616·37	623·44
38	652·04	661·47	663·46	675·57
41	695·62	701·76	710·55	717·03
44	739·21	729·13	757·64	756·27
47	782·79	768·49	804·73	797·28
50	826·37	802·06	851·82	840·76
53	869·96	850·00	898·90	894·33
56	913·54	870·03	945·99	889·63
59	957·12	850·00	993·08	946·43
62	1000·70	975·00	1040·17	990·38
65	1044·29	1100·00	1087·26	987·50
68	1087·87	941·67	1134·35	1058·33
71	1131·45	1145·83	1181·44	875·00

TABLE XXVIII. *Regression of Weeks on Quantity* (Diagrams XX and XXI)

Weeks in milk

Quantity in gallons		1911		Quantity in gallons		1912	
		Theory	Observation			Theory	Observation
175, *225*, *275*, *325*				*175*, *225*, *275*, *325*			
375	*344·7*	29·48	28·73	*375*	*330·8*	28·45	27·03
425		31·87	32·50	*425*		31·47	31·85
475		33·25	33·28	*475*		32·97	33·53
525		34·54	34·72	*525*		34·40	34·92
575		35·74	36·04	*575*		35·77	36·32
625		36·85	36·65	*625*		37·07	37·30
675		37·88	37·56	*675*		38·31	38·15
725		38·83	38·86	*725*		39·47	38·93
775		39·68	39·27	*775*		40·57	39·95
825		40·45	40·14	*825*		41·60	40·99
875		41·14	40·83	*875*		42·57	42·58
925		41·73	40·78	*925*		43·46	43·76
975		42·24	42·79	*975*		44·29	44·11
1025	*1098·3*	43·13	42·51	*1025*	*1104·0*	46·12	47·05
1075, *1125*, *1175*, *1225*, *1275*				*1075*, *1125*, *1175*, *1225*, *1275*			

TABLE XXIX. *Regression of Percentage Butter Fat on Weeks in Milk*
(Diagrams XXII and XXIII)

Percentage butter fat

Weeks in milk	1911		1912	
	Theory	Observation	Theory	Observation
8	—	—	3·66	4·88
11	3·61	3·38	3·66	—
14	3·63	3·50	3·67	3·94
17	3·64	3·73	3·68	3·67
20	3·65	3·63	3·69	3·76
23	3·67	3·72	3·70	3·74
26	3·68	3·81	3·71	3·76
29	3·69	3·69	3·71	3·72
32	3·71	3·69	3·72	3·73
35	3·72	3·70	3·73	3·71
38	3·73	3·73	3·74	3·72
41	3·75	3·77	3·75	3·75
44	3·76	3·77	3·76	3·80
47	3·77	3·81	3·76	3·78
50	3·79	3·73	3·77	3·76
53	3·80	3·81	3·78	3·78
56	3·82	3·94	3·79	3·83

TABLE XXX. *Regression of Weeks in Milk on Percentage of Butter Fat*

Weeks in milk

Percentage of butter fat	1911		1912	
	Theory	Observation	Theory	Observation
2·125	—	—	35·73	38·00
2·375	—	—	35·99	—
2·625	—	—	36·25	36·80
2·875	35·42	36·13	36·51	34·71
3·125	35·75	35·41	36·77	35·89
3·375	36·09	36·35	37·03	37·21
3·625	36·43	36·27	37·29	37·35
3·875	36·76	36·77	37·55	37·75
4·125	37·10	37·26	37·81	37·89
4·375	37·44	37·06	38·07	37·38
4·625	37·77	38·13	38·34	37·34
4·875	38·11	39·04	38·60	38·09

TABLE XXXI. *Regression of Weeks in Milk on Age*
(Diagrams XXIV and XXV)

Weeks in milk

Age	1911		Age	1912	
	Theory	Observation		Theory	Observation
4·5	35·82	35·79	*4·5*	36·39	36·27
5·5	36·09	36·06	*5·5*	36·65	36·82
6·5	36·28	36·31	*6·5*	36·88	36·92
7·5	36·39	36·54	*7·5*	37·09	37·00
8·5	36·42	36·35	*8·5*	37·27	37·32
9·5	36·35	36·34	*9·5*	37·43	37·25
10·5	36·22	36·18	*10·5*	37·57	37·58
11·5	36·00	35·50	*11·5*	37·68	37·68
12·5	35·70	36·26	*12·5*	37·77	38·06

TABLE XXXII. *Regression of Age on Weeks in Milk*

Age in years

Weeks in milk	1911		Weeks in milk	1912	
	Theory	Observation		Theory	Observation
11, 14, 17, 20	18·5		8, 11, 14, 17, 20	18·36	
20	6·03	6·47	20	5·27	5·05
23	6·34	5·01	23	5·77	5·61
26	6·47	6·26	26	6·01	5·69
29	6·54	6·79	29	6·19	6·21
32	6·55	6·52	32	6·30	6·37
35	6·49	6·52	35	6·35	6·45
38	6·38	6·44	38	6·33	6·44
41	6·21	6·06	41	6·25	6·14
44	5·97	5·97	44	6·11	5·78
47	5·68	5·41	47	5·90	5·54
50	5·13	5·48	50	5·22	5·58
53, 56	51·4		53, 56, 59, 62, 65	53·5	

TABLE XXXIII. *Regression of Yield of Butter Fat on Age*
(Diagrams XXVI and XXVII)

Yield of butter fat

Age	1911		Age	1912	
	Theory	Observation		Theory	Observation
2·5	203·31	216·10	2·5	210·85	214·10
3·5	215·59	218·27	3·5	222·50	224·82
4·5	226·33	218·32	4·5	232·98	225·42
5·5	235·52	232·49	5·5	242·28	244·22
6·5	243·17	246·51	6·5	250·40	251·40
7·5	249·28	250·18	7·5	257·34	260·51
8·5	253·84	253·54	8·5	263·11	262·72
9·5	256·86	268·17	9·5	267·69	266·32
10·5	258·34	257·76	10·5	271·10	273·31
11·5	258·27	250·35	11·5	273·33	274·40
12·5	256·66	260·11	12·5	274·38	267·08
13·5, 14·5, 15·5, 16·5, 17·5, 18·5	14·7		13·5, 14·5, 15·5, 16·5, 17·5, 18·5	14·4	
	247·69	241·24		273·12	273·39

TABLE XXXIV. *Regression of Age on Yield of Butter Fat*

Age in years

Yield of butter fat	1911		1912	
	Theory	Observation	Theory	Observation
50	4·27	3·50	—	—
70	4·49	5·50	4·29	4·14
90	4·71	4·17	4·51	4·00
110	4·93	4·45	4·73	4·36
130	5·15	5·01	4·94	4·98
150	5·37	5·26	5·16	5·11
170	5·58	5·67	5·38	5·48
190	5·80	5·93	5·60	5·64
210	6·02	5·86	5·82	5·56
230	6·24	6·17	6·04	6·02
250	6·46	6·44	6·26	6·43
270	6·68	6·87	6·48	6·58
290	6·90	7·07	6·70	6·81
310	7·12	7·30	6·92	6·91
330	7·34	7·51	7·14	7·10
350	7·56	7·06	7·35	7·20
370	7·78	7·34	7·57	7·51
390	7·99	8·09	7·79	7·50
410	8·21	5·80	8·01	8·18
430	8·43	8·00	8·23	7·72
450	8·66	9·36	8·45	7·92
470	8·88	8·06	8·67	8·31

TABLE XXXV. 1920 *Data. Age and Percentage of Butter Fat*

Age in years

Percentage butter fat Above	Up to	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	Totals
2·50	2·75	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	1
2·75	3·00	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	1
3·00	3·25	1	18	15	10	16	15	4	7	3	1	1	1	—	—	—	—	92
3·25	3·50	10	83	79	80	93	47	38	28	20	13	2	1	3	—	2	—	499
3·50	3·75	30	288	264	232	182	135	84	85	32	26	12	6	1	—	1	—	1378
3·75	4·00	37	437	414	310	226	152	99	71	44	21	12	3	1	1	—	—	1828
4·00	4·25	24	213	203	134	95	52	42	25	20	10	4	3	2	1	1	1	830
4·25	4·50	8	76	60	33	19	17	9	7	2	2	1	1	—	—	—	—	235
4·50	4·75	2	10	7	9	4	—	—	—	1	1	—	—	—	—	—	—	34
4·75	5·00	—	3	3	1	1	1	—	—	—	—	—	—	—	—	—	—	9
5·00	5·25	—	1	1	1	—	—	—	—	—	1	—	—	—	—	—	—	4
5·25	5·50	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	1
Totals		112	1129	1047	812	636	419	276	223	122	75	32	15	7	2	4	1	4912

TABLE XXXVI. 1920 *Data. Age and Weeks*

Age in years

Length of lactation period in weeks Above	Up to	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	Totals
3·50	6·50	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
6·50	9·50	—	—	1	—	—	1	—	—	—	—	—	—	—	—	—	—	2
9·50	12·50	—	1	3	—	1	—	1	1	—	—	—	—	—	—	—	—	7
12·50	15·50	—	2	3	2	—	—	1	—	—	—	—	—	—	—	—	—	8
15·50	18·50	1	4	2	7	1	—	—	1	—	—	1	—	—	—	—	—	17
18·50	21·50	4	11	7	5	2	1	—	1	—	—	—	—	1	—	—	—	32
21·50	24·50	3	21	19	17	5	11	5	1	1	—	—	—	—	—	—	—	83
24·50	27·50	2	36	31	26	26	13	13	5	7	—	—	—	—	1	—	—	160
27·50	30·50	8	70	71	53	39	23	21	13	7	3	2	1	1	—	—	—	312
30·50	33·50	15	132	114	91	90	49	33	29	18	11	1	2	1	—	1	—	587
33·50	36·50	11	170	181	141	120	72	44	46	21	16	5	4	3	—	—	1	835
36·50	39·50	21	219	194	172	127	91	57	42	22	20	10	4	—	—	—	—	979
39·50	42·50	11	206	190	139	97	76	51	48	24	17	6	2	1	1	—	—	869
42·50	45·50	20	156	122	87	70	44	24	16	15	2	5	2	—	—	2	—	565
45·50	48·50	9	75	72	42	40	23	14	16	4	3	1	—	—	—	—	—	299
48·50	51·50	6	20	28	26	12	9	10	4	3	2	1	—	—	—	—	—	121
51·50	54·50	1	4	8	2	5	4	1	—	—	1	—	—	—	—	1	—	27
54·50	57·50	—	1	1	2	1	2	1	—	—	—	—	—	—	—	—	—	8
Totals		112	1129	1047	812	636	419	276	223	122	75	32	15	7	2	4	1	4912

TABLE XXXVII. 1920 *Data. Age and Yield of Butter Fat per Week*

Age in years

Yield of butter fat per week in pounds Above	Up to	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	Totals
3·00	3·50	—	2	1	—	—	—	—	—	1	—	—	—	—	—	—	—	4
3·50	4·00	1	4	1	1	2	—	—	—	—	1	—	—	—	—	—	—	10
4·00	4·50	5	18	13	10	4	2	1	2	1	1	—	1	—	—	—	—	58
4·50	5·00	3	41	30	21	15	8	4	3	4	1	1	—	—	1	—	—	132
5·00	5·50	12	118	67	45	25	11	16	7	5	3	1	2	—	—	1	—	313
5·50	6·00	22	162	94	70	61	31	24	16	5	7	4	—	1	—	1	—	498
6·00	6·50	13	176	166	96	74	36	39	27	18	10	2	2	1	—	—	—	660
6·50	7·00	22	186	188	117	102	58	49	32	16	11	8	1	—	—	1	—	791
7·00	7·50	12	153	149	122	92	74	32	37	21	10	5	4	2	—	—	—	713
7·50	8·00	8	102	119	110	88	60	33	31	10	11	3	1	2	—	—	1	579
8·00	8·50	7	80	81	80	62	44	25	27	18	5	5	1	1	—	1	—	437
8·50	9·00	4	44	61	73	51	37	25	14	5	5	2	1	—	—	—	—	322
9·00	9·50	1	27	40	23	23	28	11	14	5	4	—	2	—	—	—	—	178
9·50	10·00	2	11	15	23	25	14	6	10	4	1	—	—	—	—	—	—	111
10·00	10·50	—	4	15	11	6	11	4	3	4	5	—	—	—	1	—	—	64
10·50	11·00	—	1	6	4	3	3	3	—	2	—	—	—	—	—	—	—	22
11·00	11·50	—	—	1	3	1	1	3	—	1	—	1	—	—	—	—	—	11
11·50	12·00	—	—	—	—	2	—	—	—	—	—	—	—	—	—	—	—	2
12·00	12·50	—	—	—	1	—	1	—	—	1	—	—	—	—	—	—	—	3
12·50	13·00	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	1
13·00	13·50	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	1
13·50	14·00	—	—	—	1	—	—	—	—	1	—	—	—	—	—	—	—	2
Totals		112	1129	1047	812	636	419	276	223	122	75	32	15	7	2	4	1	4912

TABLE XXXVIII. 1920 *Records. Age and Yield of Milk per Week*

Age in years

Yield of milk per week in gallons Above	Up to	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	Totals
7·50	8·50	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	1
8·50	9·50	—	2	2	—	1	—	—	—	—	—	—	—	—	—	—	—	5
9·50	10·50	3	5	1	1	3	—	—	—	—	—	—	—	—	—	—	—	13
10·50	11·50	2	10	8	7	1	—	1	—	2	1	—	1	—	—	—	—	33
11·50	12·50	2	25	17	9	5	4	4	2	1	1	—	—	—	1	—	—	71
12·50	13·50	9	76	29	18	9	2	4	1	1	1	—	1	—	—	—	—	151
13·50	14·50	11	76	57	38	23	9	7	6	4	2	3	—	—	—	—	—	236
14·50	15·50	11	115	79	43	34	24	11	8	4	5	1	2	1	—	1	—	339
15·50	16·50	15	149	119	74	59	23	23	16	9	7	4	—	—	—	1	—	499
16·50	17·50	16	148	131	94	58	34	32	15	12	6	5	—	1	—	—	—	552
17·50	18·50	11	146	132	83	73	49	39	22	17	6	5	1	1	—	—	—	585
18·50	19·50	10	117	112	113	87	51	35	33	11	10	2	3	1	—	—	1	586
19·50	20·50	8	97	107	79	69	51	25	30	13	10	3	3	—	—	1	—	496
20·50	21·50	3	63	93	88	70	49	31	29	9	7	4	—	1	—	1	—	448
21·50	22·50	5	42	63	49	45	32	14	18	10	3	1	2	—	—	—	—	284
22·50	23·50	1	19	33	38	38	27	17	17	12	7	1	2	2	—	—	—	214
23·50	24·50	2	20	23	34	27	19	13	9	3	2	1	—	—	—	—	—	153
24·50	25·50	3	10	15	22	17	20	8	10	3	4	—	—	—	—	—	—	112
25·50	26·50	—	7	13	7	4	15	2	4	2	3	1	—	—	—	—	—	58
26·50	27·50	—	2	7	9	5	5	4	2	—	—	—	—	—	1	—	—	35
27·50	28·50	—	—	2	1	4	2	1	1	2	—	—	—	—	—	—	—	13
28·50	29·50	—	—	2	2	4	1	3	—	3	—	—	—	—	—	—	—	15
29·50	30·50	—	—	—	—	—	2	—	—	2	—	—	—	—	—	—	—	4
30·50	31·50	—	—	2	1	—	—	2	—	—	—	—	—	—	—	—	—	5
31·50	32·50	—	—	—	2	—	—	—	—	—	—	—	—	—	—	—	—	2
32·50	33·50	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	1
33·50	34·50	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	1
Totals		112	1129	1047	812	636	419	276	223	122	75	32	15	7	2	4	1	4912

TABLE XXXIX. 1920 *Records. Percentage Butter Fat and Weeks*

Percentage butter fat

Length of lactation period in weeks Above	Up to	Above 2·50 Up to 2·75	2·75 3·00	3·00 3·25	3·25 3·50	3·50 3·75	3·75 4·00	4·00 4·25	4·25 4·50	4·50 4·75	4·75 5·00	5·00 5·25	5·25 5·50	Totals
3·50	6·50	—	—	—	—	1	—	—	—	—	—	—	—	1
6·50	9·50	—	—	1	—	1	—	—	—	—	—	—	—	2
9·50	12·50	—	—	1	—	2	1	2	1	—	—	—	—	7
12·50	15·50	—	—	—	1	1	3	1	2	—	—	—	—	8
15·50	18·50	—	—	2	1	1	4	5	3	1	—	—	—	17
18·50	21·50	—	—	1	5	11	9	3	2	—	1	—	—	32
21·50	24·50	—	—	4	9	25	20	10	3	2	—	1	—	83
24·50	27·50	—	—	4	27	41	52	20	12	2	2	—	—	160
27·50	30·50	—	—	9	37	113	95	38	15	2	2	—	1	312
30·50	33·50	—	—	15	85	172	203	87	22	3	—	—	—	587
33·50	36·50	—	—	17	97	234	313	140	28	5	1	—	—	835
36·50	39·50	—	—	12	92	268	375	172	50	8	1	1	—	979
39·50	42·50	1	—	17	79	255	316	153	43	4	1	—	—	869
42·50	45·50	—	—	3	36	135	246	106	33	5	—	1	—	565
45·50	48·50	—	1	4	22	79	109	66	14	2	1	1	—	299
48·50	51·50	—	—	1	6	26	60	22	6	—	—	—	—	121
51·50	54·50	—	—	—	1	12	9	4	1	—	—	—	—	27
54·50	57·50	—	—	1	1	1	4	1	—	—	—	—	—	8
Totals		1	1	92	499	1378	1828	830	235	34	9	4	1	4912

TABLE XL. 1920 *Data. Weeks and Yield of Butter Fat per Week*

Length of lactation period in weeks

Yield of butter fat per week in pounds Above	Up to	3·50	6·50	9·50	12·50	15·50	18·50	21·50	24·50	27·50	30·50	33·50	36·50	39·50	42·50	45·50	48·50	51·50	54·50	57·50	Totals
3·00	3·50	—	—	—	—	—	1	—	—	—	—	1	—	2	—	—	—	—	—	—	4
3·50	4·00	—	—	—	—	—	2	—	1	1	2	1	1	—	1	—	1	—	—	—	10
4·00	4·50	—	—	—	—	1	—	2	2	7	3	12	9	7	6	5	3	1	—	—	58
4·50	5·00	—	—	—	1	—	1	2	6	15	20	22	22	20	12	5	2	2	2	—	132
5·00	5·50	—	—	—	1	2	4	6	7	18	56	53	52	52	34	20	7	1	—	—	313
5·50	6·00	—	—	—	1	1	4	8	19	34	53	86	94	76	64	37	16	3	2	—	498
6·00	6·50	—	—	—	—	3	1	9	23	43	85	111	137	122	65	42	11	6	2	—	660
6·50	7·00	—	—	—	1	—	6	14	32	51	94	127	153	149	90	45	2½	5	—	—	791
7·00	7·50	—	1	—	1	1	3	5	25	44	84	128	141	134	90	37	15	4	—	—	713
7·50	8·00	—	—	1	1	2	4	8	12	32	62	126	115	96	62	38	17	3	—	—	579
8·00	8·50	—	—	1	1	1	3	7	7	26	43	69	98	71	60	36	12	1	1	—	437
8·50	9·00	1	—	2	1	3	1	8	11	17	36	46	72	62	43	13	4	1	1	—	322
9·00	9·50	—	—	—	—	—	1	6	6	12	21	24	41	34	20	7	6	—	—	—	178
9·50	10·00	—	—	3	—	1	—	6	5	4	14	16	24	23	7	6	2	—	—	—	111
10·00	10·50	—	1	—	—	1	1	2	3	5	8	9	8	12	8	5	1	—	—	—	64
10·50	11·00	—	—	—	—	—	—	—	—	3	4	3	5	6	1	—	—	—	—	—	22
11·00	11·50	—	—	—	—	—	—	—	1	—	1	—	5	1	1	2	—	—	—	—	11
11·50	12·00	—	—	—	—	—	—	—	—	—	1	—	—	—	—	1	—	—	—	—	2
12·00	12·50	—	—	—	—	1	—	—	—	—	—	—	—	—	1	1	—	—	—	—	3
12·50	13·00	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	1
13·00	13·50	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	1
13·50	14·00	—	—	—	—	—	—	—	—	—	—	1	1	—	—	—	—	—	—	—	2
Totals		1	2	7	8	17	32	83	160	312	587	835	979	869	565	299	121	27	8		4912

TABLE XLI. 1920 *Data. Weeks and Yield of Milk per week*

Length of lactation period in weeks

Yield of milk per week in gallons Above	Up to	6·50	9·50	12·50	15·50	18·50	21·50	24·50	27·50	30·50	33·50	36·50	39·50	42·50	45·50	48·50	51·50	54·50	57·50	Totals
7·50	8·50	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	1
8·50	9·50	—	—	—	—	—	—	—	—	1	1	2	—	1	—	—	—	—	—	5
9·50	10·50	—	—	—	—	1	3	—	1	2	1	1	—	1	2	1	—	—	—	13
10·50	11·50	—	—	—	1	—	—	2	—	6	2	3	7	2	5	1	4	—	—	33
11·50	12·50	—	—	—	—	—	—	1	4	7	4	17	12	13	8	3	1	—	1	71
12·50	13·50	—	—	—	1	—	2	1	8	7	25	24	27	24	10	13	6	2	1	151
13·50	14·50	—	—	—	—	2	—	4	10	11	40	44	37	34	34	14	3	3	—	236
14·50	15·50	—	—	—	—	2	5	3	9	29	30	62	59	60	43	23	11	2	1	339
15·50	16·50	—	—	—	1	2	1	10	20	31	57	73	105	81	63	40	11	3	1	499
16·50	17·50	—	—	—	1	1	4	10	11	37	74	80	113	108	62	32	12	6	1	552
17·50	18·50	—	—	1	—	—	1	11	27	38	63	104	120	94	69	37	18	2	—	585
18·50	19·50	—	—	—	—	—	5	3	16	34	74	112	112	117	58	38	13	4	—	586
19·50	20·50	—	—	—	3	2	2	9	9	29	49	94	100	98	56	24	16	3	2	496
20·50	21·50	—	—	2	1	2	5	5	17	23	47	79	97	74	62	25	9	—	—	448
21·50	22·50	—	1	—	—	—	—	5	3	14	32	52	68	44	37	19	7	2	—	284
22·50	23·50	—	—	—	—	—	2	3	7	10	37	35	50	39	22	7	1	—	1	214
23·50	24·50	—	—	—	—	1	—	5	5	13	18	23	28	31	14	8	7	—	—	153
24·50	25·50	1	—	1	—	2	2	5	6	11	14	18	19	18	9	6	—	—	—	112
25·50	26·50	—	—	2	—	2	—	4	3	7	4	2	7	16	5	4	2	—	—	58
26·50	27·50	—	1	1	—	—	—	1	3	2	7	5	6	6	1	2	—	—	—	35
27·50	28·50	—	—	—	—	—	—	—	—	—	2	2	5	1	3	—	—	—	—	13
28·50	29·50	—	—	—	—	—	—	—	—	—	5	1	4	2	2	1	—	—	—	15
29·50	30·50	—	—	—	—	—	—	—	—	—	1	—	1	1	—	1	—	—	—	4
30·50	31·50	—	—	—	—	—	—	1	—	—	—	1	—	3	—	—	—	—	—	5
31·50	32·50	—	—	—	—	—	—	—	—	—	—	1	1	—	—	—	—	—	—	2
32·50	33·50	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	1
33·50	34·50	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	1
Totals		1	2	7	8	17	32	83	160	312	587	835	979	869	565	299	121	27	8	4912

TABLE XLII. 1920 *Data. Yield of Milk per Week and Percentage of Butter Fat*

Percentage butter fat

Yield of milk per week in gallons Above	Up to	Up to 2·75	3·00	3·25	3·50	3·75	4·00	4·25	4·50	4·75	5·00	5·25	5·50	Totals
Above 2·50	2·75													
			3·00	3·25	3·50	3·75	4·00	4·25	4·50	4·75	5·00	5·25	5·50	
7·50	8·50	—	—	—	—	—	—	1	—	—	—	—	—	1
8·50	9·50	—	—	—	—	1	1	1	2	—	—	—	—	5
9·50	10·50	—	—	—	2	1	6	2	1	1	—	—	—	13
10·50	11·50	—	—	—	2	3	13	11	3	—	—	1	—	33
11·50	12·50	—	—	1	5	17	29	14	4	1	—	—	—	71
12·50	13·50	—	—	2	5	32	57	44	8	2	1	—	—	151
13·50	14·50	—	—	3	16	48	87	64	15	3	—	—	—	236
14·50	15·50	—	—	5	23	87	137	65	20	1	—	—	1	339
15·50	16·50	—	1	9	47	120	175	102	39	4	2	—	—	499
16·50	17·50	—	—	10	63	139	215	78	39	5	2	1	—	552
17·50	18·50	—	—	9	65	170	219	90	25	5	2	—	—	585
18·50	19·50	—	—	11	55	186	221	87	21	4	—	1	—	586
19·50	20·50	—	—	9	61	150	171	85	16	3	—	1	—	496
20·50	21·50	1	—	11	54	141	157	67	14	2	1	—	—	448
21·50	22·50	—	—	6	31	72	123	42	9	1	—	—	—	284
22·50	23·50	—	—	5	26	70	70	31	11	1	—	—	—	214
23·50	24·50	—	—	—	14	56	62	17	3	—	1	—	—	153
24·50	25·50	—	—	3	17	37	39	14	2	—	—	—	—	112
25·50	26·50	—	—	3	5	20	23	6	—	1	—	—	—	58
26·50	27·50	—	—	3	—	14	16	2	—	—	—	—	—	35
27·50	28·50	—	—	—	2	5	3	3	—	—	—	—	—	13
28·50	29·50	—	—	1	2	5	3	3	1	—	—	—	—	15
29·50	30·50	—	—	—	2	—	1	1	—	—	—	—	—	4
30·50	31·50	—	—	1	—	3	—	—	1	—	—	—	—	5
31·50	32·50	—	—	—	1	—	—	—	1	—	—	—	—	2
32·50	33·50	—	—	—	—	1	—	—	—	—	—	—	—	1
33·50	34·50	—	—	—	1	—	—	—	—	—	—	—	—	1
Totals		1	1	92	499	1378	1828	830	235	34	9	4	1	4912

TABLE XLIII. 1920 *Data. Month of Calving and Percentage of Butter Fat*

Percentage butter fat

Month of calving	Up to	2·75	3·00	3·25	3·50	3·75	4·00	4·25	4·50	4·75	5·00	5·25	5·50	Totals
	Above	2·50	2·75	3·00	3·25	3·50	3·75	4·00	4·25	4·50	4·75	5·00	5·25	
December		—	—	5	16	62	113	47	12	1	1	—	—	257
January		—	—	6	52	142	217	81	34	4	2	1	—	539
February		—	—	21	119	332	365	176	40	7	—	—	—	1060
March		—	1	28	159	365	478	209	51	10	2	—	—	1303
Totals—Group A		—	**1**	**60**	**346**	**901**	**1173**	**513**	**137**	**22**	**5**	**1**	—	**3159**
April		—	—	11	65	190	243	111	32	4	3	—	1	660
May		—	—	5	24	86	123	52	10	2	1	1	—	304
June		—	—	7	16	45	58	25	14	2	—	2	—	169
July		—	—	4	8	17	29	13	7	—	—	—	—	78
Totals—Group B		—	—	**27**	**113**	**338**	**453**	**201**	**63**	**8**	**4**	**3**	**1**	**1211**
August		—	—	—	7	30	30	19	5	1	—	—	—	92
September		—	—	1	12	29	35	21	8	2	—	—	—	108
October		—	—	3	8	33	65	37	15	1	—	—	—	162
November		1	—	1	13	47	72	39	7	—	—	—	—	180
Totals—Group C		**1**	—	**5**	**40**	**139**	**202**	**116**	**35**	**4**	—	—	—	**542**
Grand totals		1	1	92	499	1378	1828	830	235	34	9	4	1	4912

TABLE XLIV. 1920 *Data. Month of Calving and Age*

Age in years

Month of calving	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*	*13*	*14*	*15*	*16*	*17*	*18*	Totals
December	10	77	64	39	24	18	13	4	5	3	—	—	—	—	—	—	257
January	4	138	99	94	75	44	32	23	18	5	4	1	1	1	—	—	539
February	8	244	219	179	136	98	63	51	25	23	10	4	—	—	—	—	1060
March	16	252	268	218	191	131	79	71	36	21	9	7	1	1	1	1	1303
Totals—Group A	**38**	**711**	**650**	**530**	**426**	**291**	**187**	**149**	**84**	**52**	**23**	**12**	**2**	**2**	**1**	**1**	**3159**
April	15	155	135	102	80	48	46	37	20	10	4	1	5	—	2	—	660
May	13	78	61	44	42	22	21	5	8	7	1	2	—	—	—	—	304
June	9	41	39	21	23	11	8	7	2	5	2	—	—	—	1	—	169
July	7	8	12	16	8	11	4	7	4	—	1	—	—	—	—	—	78
Totals—Group B	**44**	**282**	**247**	**183**	**153**	**92**	**79**	**56**	**34**	**22**	**8**	**3**	**5**	—	**3**	—	**1211**
August	7	19	22	21	13	4	1	3	2	—	—	—	—	—	—	—	92
September	7	31	23	21	11	7	2	5	—	1	—	—	—	—	—	—	108
October	11	41	53	22	14	11	3	7	—	—	—	—	—	—	—	—	162
November	5	45	52	35	19	14	4	3	2	—	1	—	—	—	—	—	180
Totals—Group C	**30**	**136**	**150**	**99**	**57**	**36**	**10**	**18**	**4**	**1**	**1**	—	—	—	—	—	**542**
Grand totals	112	1129	1047	812	636	419	276	223	122	75	32	15	7	2	4	1	4912

TABLE XLV. 1920 *Records. Month of Calving and Weeks in Milk*

Weeks in milk

Month of Calving — Above / Up to	*3·50*	*6·50*	*9·50*	*12·50*	*15·50*	*18·50*	*21·50*	*24·50*	*27·50*	*30·50*	*33·50*	*36·50*	*39·50*	*42·50*	*45·50*	*48·50*	*51·50*	*54·50*	*57·50*	Totals
	6·50	*9·50*	*12·50*	*15·50*	*18·50*	*21·50*	*24·50*	*27·50*	*30·50*	*33·50*	*36·50*	*39·50*	*42·50*	*45·50*	*48·50*	*51·50*	*54·50*	*57·50*		
December	—	—	—	—	—	—	—	4	7	11	32	65	50	46	32	7	3	—		257
January	—	—	—	1	2	—	3	4	9	37	65	141	134	91	40	8	3	1		539
February	—	—	1	1	1	5	5	16	57	137	203	262	244	102	7	16	3	—		1060
March	—	1	1	2	5	4	18	42	104	215	356	299	119	52	63	19	2	1		1303
Totals—Group A	—	**1**	**2**	**4**	**8**	**9**	**26**	**66**	**177**	**400**	**656**	**767**	**547**	**291**	**142**	**50**	**11**	**2**		**3159**
April	—	—	2	1	2	9	21	54	84	146	62	31	97	89	40	18	4	—		660
May	—	—	2	1	2	7	20	30	29	6	19	56	59	41	22	5	2	3		304
June	—	—	—	1	—	2	11	1	4	10	28	36	37	27	9	3	—	—		169
July	—	1	1	—	1	2	3	—	2	1	7	9	26	6	9	6	3	1		78
Totals—Group B	—	**1**	**5**	**3**	**5**	**20**	**55**	**85**	**119**	**163**	**116**	**132**	**219**	**163**	**80**	**32**	**9**	**4**		**1211**
August	1	—	—	—	2	3	2	1	2	2	10	20	14	16	8	9	2	—		92
September	—	—	—	—	2	—	—	1	2	7	21	7	21	27	11	8	—	1		108
October	—	—	—	1	—	—	—	4	11	12	14	17	25	36	30	8	3	1		162
November	—	—	—	—	—	—	—	3	1	3	18	36	43	32	28	14	2	—		180
Totals—Group C	**1**	—	—	**1**	**4**	**3**	**2**	**9**	**16**	**24**	**63**	**80**	**103**	**111**	**77**	**39**	**7**	**2**		**542**
Grand totals	1	2	7	8	17	32	83	160	312	587	835	979	869	565	299	121	27	8		4912

TABLE XLVI. 1920 *Records. Month of Calving and Yield of Butter Fat per Week*

Month of calving

Yield of butter fat per week in pounds		December	January	February	March	Totals—Group A	April	May	June	July	Totals—Group B	August	September	October	November	Totals—Group C	Grand totals
Above	*Up to*																
3·00	*3·50*	2	—	—	1	**3**	—	—	—	—	—	1	—	—	—	**1**	4
3·50	*4·00*	2	2	2	1	**7**	2	1	—	—	**3**	—	—	—	—	—	10
4·00	*4·50*	4	4	7	8	**23**	9	4	5	2	**20**	2	3	7	3	**15**	58
4·50	*5·00*	5	9	26	31	**71**	20	14	4	8	**46**	5	—	3	7	**15**	132
5·00	*5·50*	12	22	56	80	**170**	56	27	17	7	**107**	12	5	7	12	**36**	313
5·50	*6·00*	23	52	104	117	**296**	77	34	25	6	**142**	12	15	17	16	**60**	498
6·00	*6·50*	30	76	152	176	**434**	103	38	10	9	**160**	10	14	21	21	**66**	660
6·50	*7·00*	34	78	176	239	**527**	104	47	23	11	**185**	12	19	16	32	**79**	791
7·00	*7·50*	34	76	168	195	**473**	81	46	29	6	**162**	9	15	32	22	**78**	713
7·50	*8·00*	36	67	134	145	**382**	77	30	12	13	**132**	6	18	21	20	**65**	579
8·00	*8·50*	35	65	95	119	**314**	44	15	20	3	**82**	6	5	16	14	**41**	437
8·50	*9·00*	19	32	73	95	**219**	30	22	9	6	**67**	8	6	11	11	**36**	322
9·00	*9·50*	11	21	30	49	**111**	29	8	6	2	**45**	6	3	3	10	**22**	178
9·50	*10·00*	4	18	21	29	**72**	9	11	2	3	**25**	2	3	3	6	**14**	111
10·00	*10·50*	5	9	9	13	**36**	12	3	4	1	**20**	—	1	4	3	**8**	64
10·50	*11·00*	1	4	3	2	**10**	4	2	3	—	**9**	—	1	1	1	**3**	22
11·00	*11·50*	—	2	3	2	**7**	2	—	—	—	**2**	1	—	—	1	**2**	11
11·50	*12·00*	—	—	—	—	—	1	—	—	1	**2**	—	—	—	—	—	2
12·00	*12·50*	—	1	—	1	**2**	—	—	—	—	—	—	—	—	1	**1**	3
12·50	*13·00*	—	—	—	—	—	—	1	—	—	**1**	—	—	—	—	—	1
13·00	*13·50*	—	1	—	—	**1**	—	—	—	—	—	—	—	—	—	—	1
13·50	*14·00*	—	—	1	—	**1**	—	1	—	—	**1**	—	—	—	—	—	2
Totals		257	539	1060	1303	**3159**	660	304	169	78	**1211**	92	108	162	180	**542**	4912

TABLE XLVII. 1920 *Records. Month of Calving and Yield of Milk per Week*

Month of calving

Yield of milk per week in gallons		December	January	February	March	Totals—Group A	April	May	June	July	Totals—Group B	August	September	October	November	Totals—Group C	Grand totals
Above	*Up to*																
	8·50	—	—	—	—	—	—	—	—	—	—	1	—	—	—	**1**	1
8·50	*9·50*	2	—	—	1	**3**	1	—	—	—	**1**	—	—	1	—	**1**	5
9·50	*10·50*	1	4	—	2	**7**	2	3	—	—	**5**	—	—	1	—	**1**	13
10·50	*11·50*	1	2	9	1	**13**	3	3	1	2	**9**	3	1	3	4	**11**	33
11·50	*12·50*	5	2	12	17	**36**	11	8	2	2	**23**	4	2	3	3	**12**	71
12·50	*13·50*	10	13	16	35	**74**	32	13	6	5	**56**	3	3	8	7	**21**	151
13·50	*14·50*	10	21	47	48	**126**	45	8	20	3	**76**	10	6	4	14	**34**	236
14·50	*15·50*	13	25	71	78	**187**	55	34	12	10	**111**	8	11	10	12	**41**	339
15·50	*16·50*	24	47	118	137	**326**	71	31	15	8	**125**	12	9	15	12	**48**	499
16·50	*17·50*	27	71	117	151	**366**	63	41	19	8	**131**	7	14	22	12	**55**	552
17·50	*18·50*	18	73	133	163	**387**	80	31	14	6	**131**	5	16	18	28	**67**	585
18·50	*19·50*	38	58	121	169	**386**	77	39	19	9	**144**	11	9	15	21	**56**	586
19·50	*20·50*	30	57	116	136	**339**	61	23	17	7	**108**	3	10	19	17	**49**	496
20·50	*21·50*	30	55	108	115	**308**	56	18	13	2	**89**	4	9	21	17	**51**	448
21·50	*22·50*	14	35	69	80	**198**	26	11	10	8	**55**	6	5	9	11	**31**	284
22·50	*23·50*	11	26	49	64	**150**	24	12	7	1	**44**	5	2	6	7	**20**	214
23·50	*24·50*	8	15	26	50	**99**	23	9	6	2	**40**	2	4	2	6	**14**	153
24·50	*25·50*	9	13	24	32	**78**	12	7	2	1	**22**	6	3	1	2	**12**	112
25·50	*26·50*	4	8	10	10	**32**	7	7	3	2	**19**	1	2	2	2	**7**	58
26·50	*27·50*	1	7	4	10	**22**	4	2	2	1	**9**	—	2	—	2	**4**	35
27·50	*28·50*	1	3	3	3	**10**	1	—	1	—	**2**	—	—	—	1	**1**	13
28·50	*29·50*	—	1	4	—	**5**	3	2	—	1	**6**	—	—	2	2	**4**	15
29·50	*30·50*	—	1	1	1	**3**	1	—	—	—	**1**	—	—	—	—	—	4
30·50	*31·50*	—	2	1	—	**3**	1	1	—	—	**2**	—	—	—	—	—	5
31·50	*32·50*	—	—	—	—	—	—	1	—	—	**1**	1	—	—	—	**1**	2
32·50	*33·50*	—	—	1	—	**1**	—	—	—	—	—	—	—	—	—	—	1
33·50	*34·50*	—	—	—	—	—	1	—	—	—	**1**	—	—	—	—	—	1
Totals		257	539	1060	1303	**3159**	660	304	169	78	**1211**	92	108	162	180	**542**	4912

TABLE XLVIII

Fenwick District—1920 *Data. Age and Yield per Week*

Quantity in gallons

Age	11·5	12·5	13·5	14·5	15·5	16·5	17·5	18·5	19·5	20·5	21·5	22·5	23·5	24·5	25·5	26·5	27·5	28·5	29·5	30·5	31·5	32·5	33·5	34·5	Totals
4	—	1	4	1	5	7	3	7	7	5	1	1	—	2	1	—	—	—	—	—	—	—	—	—	45
5	—	2	1	2	4	7	2	3	6	9	5	3	3	3	2	2	—	1	—	1	—	—	—	—	56
6	1	—	1	1	—	6	2	5	5	3	4	2	2	1	—	1	—	—	—	1	—	—	—	—	35
7	—	—	—	1	2	5	5	5	7	—	2	3	2	2	1	—	1	—	—	—	—	—	—	—	36
8	—	—	—	1	—	—	1	2	3	4	—	1	—	—	—	—	—	—	—	—	—	—	—	—	12
9	—	—	1	1	1	1	3	3	1	4	1	1	—	1	1	—	—	—	—	—	—	—	—	—	19
10	—	—	—	—	—	1	—	1	2	—	—	—	—	—	—	1	1	—	—	—	—	—	—	—	6
11	—	—	—	—	2	—	—	1	2	—	1	—	—	—	—	—	—	—	—	—	—	—	—	1	7
12	—	—	—	—	2	1	—	1	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	5
13	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
14	—	—	—	—	—	—	—	—	—	—	—	2	—	—	—	—	—	—	—	—	—	—	—	—	2
Totals	1	3	7	7	16	28	16	29	33	26	14	13	7	9	5	4	2	1	—	2	—	—	—	1	224

TABLE XLIX. *Distribution of Age in Years* (Diagram XLII)

Age in years	1920 data	
	Theory	Observation
2·5	148·9	112
3·5	1080·5	1129
4·5	1034·4	1047
5·5	835·3	812
6·5	624·0	636
7·5	441·3	419
8·5	297·7	276
9·5	191·8	223
10·5	117·8	122
11·5	68·6	75
12·5	37·6	32
13·5	19·3	15
14·5	9·1	7
15·5	3·8	2
16·5	1·4	4
17·5	·4	1
18·5	·1	—
Totals	4912·0	4912

TABLE L. *Distribution of Yield of Butter Fat per Week (lb.)* (Diagram XLIV)

Yield of butter fat per week (lb.)	1920 data	
	Theory	Observation
2·25	·1	—
2·75	·4	—
3·25	2·9	4
3·75	14·2	10
4·25	51·8	58
4·75	141·7	132
5·25	299·1	313
5·75	499·1	498
6·25	675·1	660
6·75	759·0	791
7·25	726·2	713
7·75	604·3	579
8·25	446·1	437
8·75	297·5	322
9·25	181·0	178
9·75	103·5	111
10·25	55·4	64
10·75	28·3	22
11·25	13·8	11
11·75	6·5	2
12·25	3·0	3
12·75	1·4	1
13·25	·6	1
13·75	·3	1
14·25	·1	1
14·75	·1	—
Totals	4911·5	4912

TABLE LII. *Distribution of Length of Lactation Period in Weeks* (Diagram XLV)

Length of lactation period in weeks	1920 data	
	Theory	Observation
Up to *5*	1·2	1
8	1·6	2
11	3·5	7
14	7·7	8
17	17·1	17
20	37·8	32
23	81·7	83
26	169·1	160
29	325·8	312
32	560·3	587
35	821·1	835
38	972·3	979
41	884·0	869
44	593·9	565
47	290·7	299
50	105·5	121
53	29·8	27
56	8·9	8
	4912·0	4912

TABLE LII. *Distribution of Average Percentage Butter Fat* (Diagram XLIII)

Average percentage butter fat	1920 data	
	Theory	Observation
2·125	—	—
2·375	·1	—
2·625	1·0	1
2·875	8·7	1
3·125	74·7	92
3·375	474·4	499
3·625	1471·7	1378
3·875	1749·0	1828
4·125	848·7	830
4·345	225·8	235
4·625	46·3	34
4·875	9·1	9
5·125	1·9	4
5·375	·4	1
5·625	·1	—
5·875	·1	—
	4912·0	4912

TABLE LIII. *Distribution of Yield of Milk per Week* (Diagram XLVI)

Yield in gallons	1920 data	
	Theory	Observation
6	·1	—
7	·3	—
8	1·2	1
9	4·2	5
10	12·6	13
11	32·6	33
12	73·4	71
13	141·8	151
14	240·8	236
15	359·4	339
16	475·2	499
17	560·0	552
18	594·4	585
19	572·8	586
20	505·8	496
21	412·7	448
22	313·6	284
23	223·8	214
24	151·0	153
25	97·0	112
26	59·7	58
27	35·4	35
28	20·3	13
29	11·3	15
30	6·2	4
31	3·3	5
32	1·7	2
33	·9	1
34	·5	1
35	·2	—
36	·1	—
	4912·3	4912

TABLE LIV (*a*). *Distribution of Yield of Milk per Week for* 3 *and* 4 *Years of Age* (Diagrams LX and LXI)

Yield of milk per week in gallons	1920 data			
	3 years		4 years	
	Theory	Observation	Theory	Observation
7	—	—	—	—
8	·1	1	·1	—
9	·3	1	1·0	2
10	1·1	1	4·2	5
11	2·6	2	13·0	10
12	5·0	2	30·5	25
13	7·8	9	57·5	76
14	10·6	11	90·1	76
15	12·6	11	121·0	115
16	13·5	15	142·2	149
17	13·3	16	148·5	148
18	12·0	11	139·4	146
19	10·0	10	119·0	117
20	7·8	8	92·9	97
21	5·7	3	66·7	63
22	3·9	5	44·3	42
23	2·5	1	27·3	19
24	1·5	2	15·6	20
25	·9	2	8·3	10
26	·5	1	4·1	7
27	·2	—	2·0	2
28	·1	—	·8	—
29	—	—	·3	—
30	—	—	·1	—
31	—	—	·1	—
	112·0	112	1129·0	1129

TABLE LIV (*b*). *Distribution of Yield of Milk per Week for* 5 *and* 6 *Years of Age*
(Diagrams LXII and LXIII)

Yield of milk per week in gallons	1920 data			
	5 years		6 years	
	Theory	Observation	Theory	Observation
6	—	—	—	—
7	—	—	—	—
8	·2	1	·2	—
9	·8	1	·6	—
10	2·4	1	1·6	1
11	6·8	8	4·1	7
12	16·0	17	9·3	9
13	32·3	29	18·6	18
14	56·1	57	33·0	38
15	84·4	79	51·7	43
16	110·9	119	71·7	74
17	128·5	131	88·7	94
18	132·7	132	98·2	83
19	123·6	112	98·2	113
20	104·9	107	89·3	79
21	82·0	93	74·5	88
22	67·7	63	57·6	49
23	40·8	33	41·6	38
24	26·4	23	28·3	34
25	16·3	15	18·3	22
26	9·7	13	11·3	7
27	5·6	7	6·7	9
28	3·1	2	3·9	1
29	1·7	2	2·2	2
30	·9	—	1·2	—
31	·5	2	·6	1
32	·2	—	·3	1
33	·1	—	·1	1
34	·1	—	·1	—
35	·1	—	·1	—
36	—	—	—	—
37	—	—	—	—
	1046·8	1047	812·0	812

TABLE LIV (*c*). *Distribution of Yield of Milk per Week for* 7 *and* 8 *Years of Age*
(Diagrams LXIV and LXV)

Yield of milk per week in gallons	1920 data			
	7 years		8 years	
	Theory	Observation	Theory	Observation
6	—	—	—	—
7	—	—	—	—
8	·2	—	—	—
9	·4	1	—	—
10	1·1	3	—	1
11	2·7	1	·3	1
12	6·0	5	1·5	2
13	12·0	9	4·7	2
14	21·5	23	10·4	9
15	34·7	34	18·7	24
16	50·1	59	28·3	23
17	64·9	58	37·6	34
18	75·6	73	44·8	49
19	79·2	87	48·6	51
20	75·0	69	48·5	51
21	64·6	70	44·8	49
22	50·9	45	38·4	32
23	36·9	38	30·7	27
24	24·8	27	22·9	19
25	15·6	17	15·9	20
26	9·2	4	10·3	15
27	5·1	5	6·2	5
28	2·7	4	3·4	2
29	1·4	2	1·7	1
30	·7	1	·8	1
31	·3	1	·3	1
32	·2	—	·1	—
33	·1	—	—	—
34	—	—	—	—
	635·9	636	418·9	419

TABLE LIV (*d*). *Distribution of Yield of Milk per Week for* 9 *and* 10 *Years of Age*
(Diagrams LXVI and LXVII)

Yield of milk per week in gallons	1920 data			
	9 years		10 years	
	Theory	Observation	Theory	Observation
9	—	—	—	—
10	·1	—	·1	—
11	·6	1	·3	1
12	1·8	4	1·0	1
13	4·5	4	2·3	1
14	9·2	7	4·8	6
15	15·8	11	8·7	8
16	23·1	23	13·6	16
17	29·4	32	19·2	15
18	32·9	39	24·2	22
19	33·2	35	27·6	33
20	30·6	25	28·6	30
21	26·0	31	26·8	29
22	20·7	14	22·7	18
23	15·6	17	17·3	17
24	11·1	13	11·9	9
25	7·6	8	7·2	10
26	5·1	2	3·9	4
27	3·3	4	1·8	2
28	2·1	1	·7	1
29	1·3	3	·2	—
30	·8	—	·1	—
31	·5	2	—	—
32	·3	—	—	—
33	·2	—	—	—
	275·8	276	223·0	223

TABLE LIV (*e*). *Distribution of Yield of Milk per Week for* 11 *and* 12 *Years and Over*
(Diagrams LXVIII and LXIX)

Yield of milk per week in gallons	1920 data			
	11 years		12 years and over	
	Theory	Observation	Theory	Observation
7	·1	—	—	—
8	·1	1	—	—
9	·2	—	·2	—
10	·5	—	·4	—
11	·9	2	·8	2
12	1·6	1	1·6	2
13	2·7	1	3·1	2
14	4·3	4	5·3	5
15	6·3	4	8·1	10
16	8·6	9	11·3	12
17	10·6	12	14·2	12
18	12·1	17	15·9	13
19	12·7	11	16·1	17
20	12·3	13	14·8	17
21	11·1	9	12·5	13
22	9·5	10	9·8	6
23	7·7	12	7·3	12
24	5·9	3	5·1	3
25	4·4	3	3·5	4
26	3·2	2	2·3	4
27	2·3	—	1·4	1
28	1·6	2	·9	—
29	1·1	3	·6	—
30	·7	2	·3	—
31	·5	—	·2	—
32	·3	—	·1	—
33	·2	—	·1	1
34	·2	1	—	—
35	·1	—	—	—
	121·8	122	135·9	136

TABLE LV. *Regression of Percentage Butter Fat on Age*
(Diagram LXXII)

Percentage butter fat

Age in years	1920 data	
	Theory	Observation
3	3·88	3·86
4	3·86	3·85
5	3·83	3·85
6	3·81	3·82
7	3·79	3·77
8	3·78	3·77
9	3·76	3·77
10	3·75	3·74
11	3·75	3·76
12	3·74	3·77
13	3·74	3·77
14	3·74	3·78
15	3·74	3·70
16	3·75	4·00
17	3·76	3·63
18	3·77	4·13

TABLE LVI. *Regression of Yield of Milk per Week on Age*
(Diagram LXXIII)

Yield per week

Age in years	1920 data	
	Theory	Observation
3	16·41	16·88
4	17·46	17·36
5	18·28	18·32
6	18·90	18·94
7	19·34	19·20
8	19·61	19·95
9	19·74	19·39
10	19·74	19·83
11	19·65	19·80
12	19·48	19·35
13	19·25	18·94
14	18·98	18·60
15	18·69	19·43
16	18·41	19·50
17	18·15	18·00
18	17·93	19·00

TABLE LVII. *Regression of Yield of Butter Fat per Week on Age*
(Diagram LXXIV)

Yield of butter fat

Age	1920 data	
	Theory	Observation
3	6·36	6·51
4	6·73	6·69
5	7·00	7·02
6	7·20	7·24
7	7·33	7·26
8	7·40	7·52
9	7·42	7·29
10	7·40	7·39
11	7·35	7·45
12	7·28	7·28
13	7·19	7·16
14	7·11	7·08
15	7·02	7·18
16	6·95	7·50
17	6·91	6·50
18	6·90	7·75

TABLE LVIII. *Regression of Weeks in Milk on Age*
(Diagram LXXV)

Weeks in milk

Age	1920 data	
	Theory	Observation
3	37·54	37·68
4	37·51	37·54
5	37·48	37·57
6	37·45	37·26
7	37·43	37·35
8	37·40	37·56
9	37·37	37·18
10	37·34	37·38
11	37·34	37·19
12	37·29	37·84
13	37·26	37·72
14	37·23	37·00
15	37·20	32·43
16	37·18	33·50
17	37·15	43·25
18	37·12	35·00

TABLE LIX. *Actual and Estimated Yields*

No. of cow	Date of calving	Date of 1st test	Total yield for lactation (in gallons) private	Total yield for lactation (in gallons) Association		Difference in yield (association *minus* private)		Date of final association test	Date of real end of lactation according to private records
				Old method	New method	Old method	New method		
1	23/4/1908	2/5/1908	602·3	621·8	647·0	+19·5	+44·7	2/3/1909	28/2/1909
2	9/4/1908	16/4/1908	805·8	810·7	339·4	+ 4·9	+33·6	30/3/1909	30/3/1909
3	10/4/1909	11/5/1909	606·1	603·2	624·9	− 2·9	+18·8	4/2/1910	14/2/1910
4	13/4/1910	29/4/1910	625·4	630·6	652·3	+ 5·2	+26·9	21/2/1911	24/2/1911
5	16/4/1909	11/5/1909	736·6	732·0	753·4	− 4·6	+16·8	4/2/1910	16/2/1910
6	9/4/1910	15/4/1910	786·1	787·4	813·3	+ 1·3	+27·2	7/2/1911	9/2/1911
7	22/4/1911	1/5/1911	751·0	754·7	782·0	+ 3·7	+31·0	22/1/1912	27/1/1912
8	19/4/1909	11/5/1909	742·1	745·1	768·2	+ 3·0	+26·1	15/4/1910	15/4/1910
9	16/4/1910	29/4/1910	884·2	885·8	910·0	+ 1·6	+25·8	7/2/1911	13/2/1911
10	12/4/1911	1/5/1911	1015·0	1004·2	1038·2	−10·8	+23·2	22/1/1912	1/2/1912
11	11/4/1910	15/4/1910	676·1	661·9	677·7	−14·2	+ 1·6	7/2/1911	10/2/1911
12	14/4/1911	1/5/1911	656·7	652·1	672·8	− 4·6	+16·1	18/1/1912	13/1/1912
13	23/4/1910	29/4/1910	735·4	734·5	756·2	− ·9	+20·8	20/3/1911	30/3/1911
14	18/4/1910	29/4/1910	827·1	831·5	858·8	+ 4·4	+31·7	6/1/1911	19/1/1911
15	11/4/1911	1/5/1911	651·3	659·9	680·9	+ 8·6	+29·6	19/2/1912	25/2/1912
16	10/4/1912	29/4/1912	839·7	845·1	866·1	+ 5·4	+26·4	7/7/1913	9/7/1913
17	9/4/1912	29/4/1912	809·9	799·5	826·1	−10·4	+16·2	20/1/1913	22/1/1913
18	16/4/1912	29/4/1912	739·3	763·0	785·4	+23·7	+46·1	20/1/1913	26/1/1913
19	22/4/1909	27/4/1909	742·5	737·4	765·1	− 5·1	+22·6	21/1/1910	29/1/1910
20	15/4/1910	29/4/1910	786·3	792·1	817·3	+ 5·8	+31·0	24/1/1911	27/1/1911
21	11/4/1910	15/4/1910	827·4	810·2	840·7	−17·2	+13·3	23/12/1910	26/12/1910
22	4/4/1911	17/4/1911	884·1	866·4	892·0	−17·7	+ 7·9	25/12/1911	30/12/1911
23	30/4/1909	11/5/1909	803·3	795·1	826·6	− 8·2	+23·3	7/1/1910	15/1/1910
24	10/4/1910	15/4/1910	620·9	608·3	638·4	−12·6	+17·5	11/11/1910	14/11/1910
25	18/4/1909	27/4/1909	856·5	850·5	870·8	− 6·0	+14·3	7/1/1910	15/1/1910
26	11/4/1909	27/4/1909	963·4	990·3	1010·3	+26·9	+46·9	18/2/1910	24/2/1910
27	17/4/1909	27/4/1909	800·5	798·5	821·6	− 2·0	+21·1	18/2/1910	26/2/1910
28	29/4/1910	13/5/1910	782·4	785·0	811·3	+ 2·6	+28·9	21/2/1911	22/2/1911
29	23/4/1911	1/5/1911	760·7	765·9	793·6	+ 5·2	+32·9	5/2/1912	6/2/1912
30	12/4/1912	15/4/1912	709·0	729·8	761·3	+20·8	+52·3	9/12/1912	17/12/1912
31	4/4/1911	17/4/1911	848·6	862·5	886·3	+13·9	+37·7	8/1/1912	9/1/1912
32	6/4/1911	17/4/1911	594·5	605·2	630·8	+10·7	+36·3	11/12/1911	14/12/1911
33	5/4/1910	15/4/1910	1105·4	1089·5	1114·0	−15·9	+ 8·6	6/1/1911	7/1/1911
34	24/4/1911	1/5/1911	770·9	767·2	793·8	− 3·7	+22·9	20/1/1912	26/1/1912
35	11/4/1912	29/4/1912	802·1	803·4	827·2	+ 1·3	+25·1	9/12/1912	16/12/1912
		Totals	27148·6	27180·3	28053·8	+31·7	905·2		
		Mean	775·7	776·6	801·5	·906	25·86		

TABLE LX. *Mean Yield (in Gallons) per cow for each Fortnight*

Fortnightly interval	(1)	(2)	(3)	(4)	Differences between (1) and (2)	Differences between (3) and (4)
	Old method		New method			
	Private	Association	Private	Association		
2nd	50·71	52·90	55·04	54·85	+2·19	− ·19
3rd	55·50	59·56	58·40	60·28	+4·06	+1·88
4th	58·31	57·87	57·53	57·40	− ·44	− ·13
5th	56·54	55·65	54·25	54·36	− ·89	+ ·11
6th	52·88	51·18	51·09	51·66	−1·70	+ ·57
7th	49·24	49·04	47·72	48·48	− ·20	+ ·76
8th	47·16	46·28	45·69	46·37	− ·88	+ ·68
9th	45·18	43·92	44·03	44·35	−1·26	+ ·32
10th	40·98	39·82	39·08	40·33	−1·16	+1·25
11th	37·00	36·06	35·25	35·75	− ·94	+ ·50
12th	33·80	33·44	32·17	32·39	− ·36	+ ·22
13th	32·12	31·62	31·02	31·62	− ·50	+ ·60
14th	30·59	30·68	29·63	30·20	+ ·09	+ ·57
15th	28·62	28·14	27·87	28·23	− ·48	+ ·36
16th	26·67	25·40	25·62	25·94	−1·27	+ ·32
17th	24·28	23·08	23·25	23·38	−1·20	+ ·13
18th	22·69	21·93	22·50	22·75	− ·76	+ ·25
19th	20·70	19·19	20·41	21·05	−1·51	+ ·64
20th	17·74	16·90	18·04	18·38	− ·84	+ ·34
21st	16·84	16·12	15·52	16·09	− ·72	+ ·57
22nd	14·95	14·13	17·16	17·48	− ·82	+ ·32

TABLE LXI

Actual Yields (from Private Records, and Estimated Yields from Association Records) for first fortnightly period, the start of the fortnight being taken as prescribed (1) *by the Old Method and* (2) *by the New Method respectively*

No. of cow	Private		Association		(5)	(6)
	(1)	(2)	(3)	(4)		
	Actual yield for 1st period		Estimated yield for 1st period		Difference between (1) and (3) (gallons)	Difference between (2) and (4) (gallons)
	Old method (gallons)	New method (gallons)	Old method (gallons)	New method (gallons)		
1	7·1	33·1	32·4	57·6	+25·3	+24·5
2	108·8	140·9	123·0	151·7	+14·2	+10·8
3	71·0	98·2	96·1	117·8	+25·1	+19·6
4	36·7	66·9	49·6	71·3	+12·9	+ 4·4
5	63·4	94·6	76·2	97·6	+12·8	+ 3·0
6	18·8	42·5	22·2	48·1	+ 3·4	+ 5·6
7	26·5	54·8	35·1	62·4	+ 8·6	+ 7·6
8	54·3	72·2	72·6	95·7	+18·3	+23·5
9	26·3	58·8	44·8	69·0	+18·5	+10·2
10	68·3	101·1	92·1	131·0	+23·8	+29·9
11	0*	16·3	9·0	24·8	+ 9·0	+ 8·5
12	48·7	66·5	50·1	70·8	+ 1·4	+ 4·3
13	10·6	31·5	18·6	40·3	+ 8·0	+ 8·8
14	29·0	57·1	42·9	70·2	+13·9	+13·1
15	43·2	71·3	60·0	81·0	+16·8	+ 9·7
16	54·5	76·7	57·0	78·0	+ 2·5	+ 1·3
17	68·3	93·4	76·0	106·4	+ 7·7	+13·0
18	32·2	52·6	41·6	64·0	+ 9·4	+11·4
19	8·8	32·5	14·7	35·4	+ 5·9	+ 2·9
20	38·3	63·5	50·4	75·6	+12·1	+12·1
21	8·5	35·7	17·4	47·9	+ 8·9	+12·2
22	45·2	73·9	47·4	73·0	+ 2·2	− ·9
23	31·9	64·5	49·5	81·0	+17·6	+16·5
24	15·3	42·9	21·5	51·6	+ 6·2	+ 8·7
25	19·3	43·9	26·1	46·4	+ 6·8	+ 2·5
26	23·4	49·5	45·6	65·6	+22·2	+16·1
27	21·6	50·5	33·0	56·1	+11·4	+ 5·6
28	42·9	68·1	52·5	78·8	+ 9·6	+10·7
29	24·1	49·5	31·6	59·3	+ 7·5	+ 9·8
30	0*	19·6	13·5	45·0	+13·5	+25·4
31	30·1	55·9	44·2	68·0	+14·1	+12·1
32	19·8	55·1	40·1	65·7	+20·3	+10·6
33	26·0	52·5	35·0	59·5	+ 9·0	+ 7·0
34	19·0	47·0	26·6	53·2	+ 7·6	+ 6·2
35	50·2	74·4	61·2	85·0	+11·0	+10·6
Totals	1192·1	2107·5	1609·6	2484·8		
Averages	34·06	60·21	45·99	70·97		

* Private record starts after date of 1st Association test.

TABLE LXII

No. of cow	Total yield in gallons		Difference Association—Private	No. of cow	Total yield in gallons		Difference Association—Private
	Private	Association			Private	Association	
1	713·3	713·3	0	*57*	709·0	729·8	+20·8
2	652·3	622·4	−29·9*	*58*	415·7	399·7	−16·0
3	595·6	596·3	+ ·7	*59*	891·3	906·4	+15·1
4	711·1	723·0	+11·9	*60*	885·6	879·1	− 6·5
5	602·3	621·8	+19·5	*61*	848·6	862·5	+13·9
6	805·8	810·7	+ 4·9	*62*	954·6	963·1	+ 8·5
7	606·1	603·2	− 2·9	*63*	743·6	736·0	− 7·6
8	625·4	630·6	+ 5·2	*64*	676·7	733·8	+57·1†
9	616·7	609·3	− 7·4	*65*	806·7	791·4	−15·3
10	736·6	732·0	− 4·6	*66*	909·9	911·9	+ 2·0
11	786·1	787·4	+ 1·3	*67*	915·5	924·5	+ 9·0
12	751·0	754·7	+ 3·7	*68*	933·9	910·6	−23·3
13	935·3	934·7	− ·6	*69*	907·9	913·5	+ 5·6
14	742·1	745·1	+ 3·0	*70*	1009·2	1021·5	+12·3
15	884·2	885·8	+ 1·6	*71*	771·4	765·2	− 6·2
16	1015·0	1004·2	−10·8	*72*	821·1	841·6	+20·5
17	676·7	661·9	−14·8	*73*	682·3	695·0	+12·7
18	656·7	652·1	− 4·6	*74*	718·1	720·3	+ 2·2
19	735·4	734·5	− ·9	*75*	652·4	652·7	+ ·3
20	677·4	655·6	−21·8	*76*	665·6	656·9	− 8·7
21	827·1	831·5	+ 4·4	*77*	594·5	605·2	+10·7
22	785·8	796·7	+10·9	*78*	694·6	751·1	+56·5
23	685·8	680·1	− 5·7	*79*	744·0	736·0	− 8·0
24	767·0	791·9	+24·9	*80*	759·3	743·9	−15·4
25	651·3	659·9	+ 8·6	*81*	1042·5	1025·4	−17·1
26	713·7	735·7	+22·0	*82*	828·7	828·5	− ·2
27	839·7	845·1	+ 5·4	*83*	803·3	784·6	−18·7
28	809·9	799·5	−10·4	*84*	1105·4	1089·5	−15·9
29	700·6	707·2	+ 6·6	*85*	593·0	595·7	+ 2·7
30	671·0	669·6	− 1·4	*86*	1411·4	1415·7	+ 4·3
31	739·3	763·0	+23·7	*87*	749·5	754·1	+ 4·6
32	536·8	568·6	+31·8	*88*	903·0	905·9	+ 2·9
33	701·0	697·2	− 3·8	*89*	957·2	978·5	+21·3
34	966·4	972·0	+ 5·6	*90*	720·6	720·0	− ·6
35	962·4	953·8	− 8·6	*91*	957·2	958·4	+ 1·2
36	1029·1	1022·4	− 6·7	*92*	786·2	776·8	− 9·4
37	689·7	703·3	+13·6	*93*	770·9	767·2	− 3·7
38	621·4	611·3	−10·1	*94*	751·7	752·0	+ ·3
39	584·0	628·4	+44·4	*95*	726·4	727·0	+ ·6
40	716·4	716·2	− ·2	*96*	906·4	902·7	− 3·7
41	742·5	737·4	− 5·1	*97*	931·2	919·0	−12·2
42	786·3	792·1	+ 5·8	*98*	846·5	840·2	− 6·3
43	827·4	810·2	−17·2	*99*	780·9	776·0	− 4·9
44	884·1	866·4	−17·7	*100*	802·1	803·4	+ 1·3
45	996·8	1003·0	+ 6·2	*101*	848·1	820·0	−28·1
46	803·3	795·1	− 8·2	*102*	1013·6	1005·4	− 8·2
47	620·9	608·3	−12·6	*103*	911·8	901·6	−10·2
48	856·5	850·5	− 6·0	*104*	882·0	886·9	+ 4·9
49	913·3	917·0	+ 3·7	*105*	690·5	700·4	+ 9·9
50	899·6	891·4	− 8·2	*106*	712·8	726·9	+14·1
51	910·4	903·5	− 6·9	*107*	626·1	645·2	+19·1
52	908·2	908·8	+ ·6				
53	963·4	990·3	+26·9		84750·3	84914·1	+163·8
54	800·5	798·5	− 2·0				
55	782·4	785·0	+ 2·6	Mean per cow	792·06	793·59	+ 1·53
56	760·7	765·9	+ 5·2				

* Maximum defect. † Maximum excess.

TABLE LXIII

Actual and Estimated Yields—24 Cows (M'Candlish and M'Vicar's Data)

(The estimated yields determined from tests every 10 days in accordance with the Association's method.)

No.	Actual yield (gallons)	Estimated yield (old method) (gallons)	Difference (gallons)	Estimated yield (new method) (gallons)	Difference (gallons)
1	702·25	706·0	+ 3·75	724·0	+ 21·75
2	621·4	622·65	+ 1·25	636·9	+ 15·50
3	747·1	764·85	+ 17·75	785·6	+ 38·5
4	726·3	727·95	+ 1·65	742·7	+ 16·4
5	528·25	524·8	− 3·45	537·8	+ 9·55
6	722·75	740·75	+ 18·0	754·0	+ 31·25
7	463·2	481·7	+ 18·50	506·2	+ 43·0
8	682·95	685·9	+ 2·95	710·4	+ 27·45
9	403·9	414·4	+ 10·50	430·15	+ 26·25
10	539·65	539·2	− ·45	553·7	+ 14·05
11	706·1	701·5	− 4·6	724·0	+ 17·9
12	862·55	862·4	− ·15	880·4	+ 17·85
13	513·7	507·45	− 6·25	527·2	+ 13·5
14	515·6	525·7	+ 10·10	539·7	+ 24·1
15	756·05	762·75	+ 6·7	778·0	+ 21·95
16	608·95	604·9	− 4·05	623·4	+ 14·45
17	552·8	559·5	+ 6·7	572·0	+ 19·2
18	767·85	770·0	+ 2·15	788·75	+ 20·9
19	867·9	859·75	− 8·15	878·5	+ 10·6
20	610·4	607·3	− 3·1	626·3	+ 15·9
21	668·5	679·2	+ 10·7	695·95	+ 27·45
22	459·65	445·0	− 14·65	461·25	+ 1·6
23	679·3	676·2	− 3·10	697·70	+ 18·4
24	719·4	717·8	− 1·6	737·3	+ 17·9
Averages	642·8	637·3	− 5·5	661·9	+ 19·1

TABLE LXIV

Actual and Estimated Yields—24 Cows (M'Candlish and M'Vicar's Data)

(The estimated yields determined from fortnightly tests in accordance with the Association's method.)

No.	Actual yields (gallons)	Estimated yields (old method) (gallons)	Difference (gallons)	Estimated yields (new method) (gallons)	Difference (gallons)
1	702·25	692·90	− 9·35	718·8	+ 16·55
2	621·4	617·55	− 3·85	636·8	+ 15·4
3	747·1	765·4	+ 18·3	795·5	+ 48·4
4	726·3	714·4	− 11·9	733·3	+ 7·0
5	528·25	503·4	− 24·85	525·8	− 2·45
6	722·75	719·6	− 3·15	736·75	+ 14·0
7	463·2	482·3	+ 19·1	516·6	+ 53·4
8	682·95	671·5	− 11·45	700·55	+ 17·6
9	403·9	410·0	+ 6·1	435·2	+ 31·3
10	539·65	520·4	− 19·25	540·7	+ 1·05
11	706·1	702·1	− 4·0	732·9	+ 26·8
12	862·55	843·45	− 19·1	869·0	+ 6·45
13	513·7	496·8	− 16·9	525·5	+ 11·8
14	515·6	517·25	+ 1·65	538·6	+ 23·0
15	756·05	750·3	− 5·75	772·0	+ 15·95
16	608·95	591·4	− 17·55	613·1	+ 4·15
17	552·8	555·65	+ 2·85	573·5	+ 20·7
18	767·85	743·25	− 24·6	768·1	+ ·25
19	867·9	861·7	− 6·2	888·65	+ 20·75
20	610·4	605·5	− 4·9	634·9	+ 24·5
21	668·5	670·2	+ 1·7	693·65	+ 25·15
22	459·65	454·05	− 5·6	477·5	+ 17·85
23	679·3	683·8	+ 4·5	711·1	+ 31·8
24	719·4	722·6	+ 3·2	747·8	+ 28·4
Averages	642·8	645·3	+ 2·5	663·0	+ 20·2

TABLE LXV. *Results of Daily Records of* 18 *Cows and corresponding Association's Estimates*

No.	Date of opening of record	Actual yield Private	Association's estimate			
			Old method	Difference	New method	Difference
1	19/11/16	660·1	645·7	−14·4	669·0	+ 8·9
2	1/8/15	562·5	594·8	+32·3	617·0	+54·5
3	21/9/16	759·0	729·0	−30·0	757·0	− 2·0
4	4/1/17	572·6	576·1	+ 3·5	599·0	+26·4
5	29/4/14	892·2	869·8	−22·4	903·7	+11·5
6	6/7/15	911·4	944·5	+33·1	977·0	+65·6
7	11/9/16	855·5	855·6	+ ·1	888·0	+32·5
8	7/4/14	947·9	934·0	−13·9	967·9	+20·0
9	18/7/15	1064·2	1109·5	+45·3	1147·0	+82·8
10	6/10/16	989·1	963·7	−25·4	1000·0	+10·9
11	6/4/17	453·7	451·4	− 2·3	475·0	+21·3
12	6/4/16	586·1	588·1	+ 2·0	611·0	+24·9
13	9/4/14	783·5	769·4	−14·1	797·0	+13·5
14	2/7/15	692·3	681·8	−10·5	707·0	+14·7
15	23/9/16	337·5	344·4	+ 6·9	365·0	+27·5
16	20/4/16	505·6	515·4	+ 9·8	539·0	+33·4
17	25/11/16	697·7	658·7	−39·0	683·0	−14·7
18	24/5/16	486·8	501·6	+14·8	521·0	+34·2
Averages		708·8	707·4	− 1·4	734·7	25·9

TABLE LXVI

Weeks in milk	Percentage frequency	
	1920 Ayrshires S.M.R. data	Interdepartmental
2	0·0	11·9
5	0·0	9·7
8	0·0	9·5
11	0·1	10·1
14	0·2	7·8
17	0·4	7·3
20	0·7	5·6
23	1·7	6·6
26	3·3	6·6
29	6·4	7·3
32	11·9	5·4
35	17·0	5·0
38	19·9	5·0
41	17·7	2·2
44	11·5	—
47	6·1	—
50	2·4	—
53	0·5	—
56	0·2	—
	100·0	100·0

TABLE LXVII. *Regression of Yield on Age* (Diagram XLI B)

Age	1912		Age	1923	
	Theory	Observed		Theory	Observed
4	18·93	18·10	*4*	20·04	19·53
5	19·21	19·23	*5*	20·17	20·30
6	19·50	20·14	*6*	20·30	20·38
7	19·79	20·36	*7*	20·43	20·71
8	20·07	19·69	*8*	20·55	20·80
9	20·36	21·34	*9*	20·68	20·44
10	20·65	20·69	*10*	20·81	21·19
11	20·94	19·63	*11*	20·94	21·05
12	21·22	20·50	*12*	21·07	21·16
13	21·51	21·64	*13*	21·20	19·57
14	21·80	22·17	*14*	21·33	20·50
15·8	22·31	18·90	*15·5*	21·52	18·69

TABLE LXVIII. *Regression of Yield on Age* (Diagram XLI A)

Yield per milking in pounds

Age	Interdepartmental		Age	1920 Ayrshires	
	Theory	Observed		Theory	Observed
3	9·98	10·33	*3*	11·84	12·41
4	10·16	10·75	*4*	12·48	12·36
5	10·34	9·47	*5*	13·02	13·06
6	10·52	9·88	*6*	13·46	13·69
7	10·70	11·27	*7*	13·80	13·67
8	10·88	11·80	*8*	14·05	14·28
9·89	11·21	10·84	*9*	14·19	13·88
			10	14·24	14·09
			11	14·19	14·07
			12	14·04	13·88
			13	13·80	13·69
			14	13·45	13·24
			15	13·01	13·45
			16	12·47	13·93
			17	11·83	11·43
			18	11·09	13·57

TABLE LXIX. *Number of Cows and Cell Frequencies for each Year of Age and for each Number of Weeks Sub-range*

Weeks in milk	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	Totals
26	8	43	29	28	15	11	10	11	2	3	160
29	21	61	71	48	50	41	31	19	19	12	373
32	32	134	119	118	97	75	68	45	38	26	752
35	27	245	232	186	191	138	137	95	63	32	1346
38	23	240	224	228	174	120	134	100	65	27	1335
41	39	272	126	112	123	88	82	60	37	28	967
44	25	172	65	59	47	47	45	21	17	16	514
47	22	91	34	30	32	18	15	10	12	9	273
50	11	59	15	20	11	10	11	8	2	3	150
Totals	208	1317	915	829	740	548	533	369	255	156	5870

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